ELD STANDARD 3: The Language of Mathematics	MAISA Unit Grade, Unit 1, Title: Function Relationships

CONNECTIONS: Michigan Academic Standards for Mathematics - Algebra I

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students listen to a narrative read aloud and graph the relationship between independent and dependent variables. Rich narratives allow students to interpret details of the text in a variety of ways. In the example illustrating the listening strand below, students are not told Kendra's driving rate either before or after she is given a ticket. As such, graphs that students produce will show a diversity of thinking. Allowing students to compare diverse graphs in a variety of contexts (e.g., partners, small group, whole group) gives them the opportunity to both build deeper mathematical understanding and strengthen their language skills. In order to illustrate the story, the teacher could either use images found on the internet as seen in the supports or sketch images as described in the contexts. An example of a sketched image is also shown in the supports for the reading strand for this unit.

The example in the strand below was taken from Mathematics Teacher, September 1994 Vol. 87, No.6, NCTM (438). As written, the problem directs students to graph time on the horizontal axis and distance on the vertical axis. Alternatively, the teacher could facilitate a dialogue where students, first individually or in small groups, make sense of the context and identify variables that could be modeled with this story. After indicating independent and dependent variables using language like, "distance depends on time" or "speed depends on time", students can identify which variable will be graphed on each axis. Teachers can facilitate conversations with prompts and questions like, "What are the two variables in the story? Which variable should go on which axis [point to anchor chart]? Why?" This alternative helps students build the habit of making sense of problems and attending to precision.

COGNITIVE FUNCTION: Students at all levels of English language proficiency INTERPRET narratives in order to represent mathematical relationships graphically.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Interpret a fully illustrated linguistically complex mathematical description read aloud by the teacher who purposefully pauses during the first read aloud and rereads the description a second time, gesturing to illustrations, in order to represent the relationship in a graph using a pictures that fully illustrate the problem while working in a small group with other students at levels 1 and 2. E.g., First read through: "Kendra is speeding [pause] along the highway [pause] and is stopped by a police officer [pause]. The officer gives her a ticket [pause] and then she continues on her way [pause]. Graph time [pause] on the horizontal axis [pause] and her speed [pause] on the vertical axis."	Interpret a fully illustrated linguistically complex mathematical description read aloud by the teacher who purposefully pauses during the first read aloud rereads the description a second time, gesturing to illustrations, in order to represent the relationship in a graph using pictures that fully illustrate the problem while working in a small group with other students at levels 1 and 2. E.g., First read through: "Kendra is speeding along the highway [pause] and is stopped by a police officer [pause]. The officer gives her a ticket [pause] and then she continues on her way [pause]. Graph time [pause] on the horizontal axis [pause] and her speed [pause] on the vertical axis."	Interpret a linguistically complex mathematical description read aloud by the teacher, who purposefully pauses, in order to represent the relationship in a graph using an illustrated reference sheet for contextual words and checking work with a partner. E.g., "Kendra is speeding along the highway [pause] and is stopped by a police officer [pause]. The officer gives her a ticket [pause] and then she continues on her way [pause]. Graph time [pause] on the horizontal axis [pause] and her speed [pause] on the vertical axis."	Interpret a linguistically complex mathematical description read aloud by the teacher, who purposefully pauses, in order to represent the relationship in a graph, checking work with a partner. E.g., "Kendra is speeding along the highway [pause] and is stopped by a police officer [pause]. The officer gives her a ticket [pause] and then she continues on her way [pause]. Graph time [pause] on the horizontal axis [pause] and her speed [pause] on the vertical axis."	Interpret a linguistically complex mathematical description read aloud by the teacher, who purposefully pauses, in order to represent the relationship in a graph, checking work with a partner. E.g., "Kendra is speeding along the highway [pause] and is stopped by a police officer [pause]. The officer gives her a ticket [pause] and then she continues on her way [pause]. Graph time [pause] on the horizontal axis [pause] and her speed [pause] on the vertical axis."	

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	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Listening Continued	Second read through: "Kendra is speeding [pause and point to a picture of a speed limit sign and speedometer] along the highway [pause and poing to a picture of a highway] and is stopped by a police officer [pause and point to a picture of a police office standing by a car]. The officer gives her a ticket [pause and point to a picture of a police officer giving a ticket] and then she continues on her way [pause and point to a picture of cars on the highway]. Graph time [pause] on the horizontal axis [pause and refer students to their anchor chart and her speed [pause] on the vertical axis [refer students to their anchor chart]."	Second read through: "Kendra is speeding [pause and point to a picture of a speed limit sign and speedometer] along the highway [pause and point to a picture of a highway] and is stopped by a police officer [pause and point to a picture of a police office standing by a car]. The officer gives her a ticket [pause and point to a picture of a police officer giving a ticket] and then she continues on her way [pause and point to a picture of cars on the highway]. Graph time [pause] on the horizontal axis [pause and refer students to their anchor chart and her speed [pause] on the vertical axis [refer students to their anchor chart]."				

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will generate, interpret, and justify qualitative features of graphs based on specific details in given contexts. Graphs can be generated from many different narratives including written sources, spoken text, and videos.

The example in the strand below describes the distance from the ground of a man crossing a bridge as seen on video #1 from: https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2083. Students graph the scenario shown in the video and refine their work after watching the same scenario at half-speed. They should have conversations with a partner or group using appropriate mathematical and contextual language before they then check their graphical representations given at the end of the video. There are ten videos that could be used in a similar way. Teachers might use one of the videos to model both mathematical content and language usage.

An example of a student describing graphical features related to a context can be found at: https://www.youtube.com/watch?v=WRTLJ1CvSKw.

COGNITIVE FUNCTION: Students at all levels of English language proficiency DESCRIBE to a partner how a graph of a function relates to visual or written narratives in various contexts.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking	Describe to a partner in short phrases the key features of a graph and how it relates to a story using sentence stems with word choices, an anchor chart illustrating word choices (e.g., increasing, decreasing, not changing), and/or gestures to demonstrate his/her thinking by pointing to the graph and another representation of the story (e.g., parts of a video, pictures of an illustrated story) The graph is _____ (increasing/decreasing/not changing) at the _____(beginning/middle/ end) of the story (points to graph for reference). [This same frame can be used for each segment of the graph.]	Describe to a partner in short sentences, using transition words, the key features of a graph and how it relates to a story using sentence frames with word choices, an anchor chart including word choices (e.g., increasing, decreasing, not changing), and pointing to the graph. The graph is _____(increasing/ decreasing/not changing) at the _____ (beginning/ middle/end) of the story (points to graph for reference) when_____[e.g., the man is walking up the bridge]. [This same frame can be used for each segment of the graph.]	Describe to a partner in multiple complete sentences using transition words, the key features of a graph and how it relates to a story using sentence frames and a suggested word list (e.g., increases/increasing, decreases/decreasing, horizontal axis, vertical axis, no change/remains constant) while pointing to the graph. First, the video shows_____. The graph shows this when_____ Next, the video shows_____. The graph shows this when_____ Last, the video shows_____. The graph shows this when_____	Describe to a partner using compound and/or complex sentences that include transition words the key features of a graph and how it relates to a storypointing to their own graph as they describe it using a suggested word list (e.g., increases/increasing, decreases/decreasing, horizontal/x-axis, vertical/y- axis, no change/remains constant). E.g., "I graphed time on the x-axis and elevation on the y-axis. As the man starts crossing the bridge, his height from the ground gradually increases. Next, his height shows no change when he pauses. The graph begins to decrease when he passes the highest part of the bridge."	Describe to a partner using compound and/or complex sentences that include transition words the key features of graph and how it relates to a story pointing to their own graph as they describe it using a required word list (e.g., increases/increasing/increased, decreases/decreasing, horizontal/x-axis, vertical/y-axis, no change/remains constant). E.g., "I graphed time on the x-axis and elevation on the y-axis. As the man started crossing the bridge, his height from the ground gradually increased. Next, his height showed no change when he paused. The graph began to decrease when he passed the highest part of the bridge."	

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students read a narrative containing a mathematical relationship and design a graph, or choose from several graphs, the one that best represents the relationship between the independent and dependent variables. Students identify from the narrative which variables should be labeled on the horizontal and vertical axes and which intervals on the graph should represent the dependent variable as increasing, decreasing, or no change, in relation to the independent variable. To increase the complexity of the mathematics, the graphs might also include an appropriate number scale if specific number values were used in the narrative. Teachers can use the graphs that students produce to formatively measure students' reading comprehension related to and conceptual understanding of the relationship between independent and dependent variables.

The strand below includes content taught at the beginning of Algebra 1. At this point in the course, students are just beginning to make sense of the significance of graphical features (e.g., slope, intercepts, local maximum and minimum, intervals where the function is increasing or decreasing). In order to support students in building their facility with connecting verbal and graphical representations while building on the foundational skills related to graphical features, the directions in the example below explicitly tell students which variable to put on each axis. This is in intentional and can be gradually removed as students build proficiency.

COGNITIVE FUNCTION: Students at all levels of English language proficiency INTERPRET mathematical relationships found in written narratives in order to design or identify an appropriate graphical representation.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Interpret a glossed version of a linguistically complex mathematical description in order to represent the relationship in a graph (or choose the best answer from a group of graphs), with key words and phrases highlighted or underlined by the teacher, using an anchor chart (e.g., horizontal axis, vertical axis), and working with a partner. E.g., "Sara walks from her home to the store. [gloss with a sketch of a stick figure walking on a line from home to a store] Halfway to the store [gloss with a sketch of a stick figure arrow pointing halfway to the store], she realizes [gloss: remembers] that she forgot to bring money,	Interpret a glossed version of a linguistically complex mathematical description in order to represent the relationship in a graph (or choose the best answer from a group of graphs) when instructed by the teacher to underline or highlight key words and phrases in the description, using an anchor chart for mathematical terms (e.g., horizontal axis, vertical axis), and working with a partner. E.g., "Sara walks from her home to the store. [gloss with a sketch of a stick figure walking on a line from home to a store] Halfway to the store [gloss with a sketch of a stick figure arrow pointing halfway to the store],	Interpret a linguistically complex mathematical description in order to represent the relationship in a graph (or choose the best answer from a group of graphs), when instructed by the teacher to underline or highlight key words and phrases in the description, using an anchor chart for mathematical terms (e.g., horizontal axis, vertical axis), an illustrated word bank for contextual words (e.g., store, money, halfway) and working with a partner. E.g., "Sara walks from her home to the store. Halfway to the store, she realizes that she forgot to bring money, so she turns around, returns home, gets her money, and then walks all the way to the store.	Interpret a linguistically complex mathematical description in order to represent the relationship in a graph (or choose the best answer from a group of graphs), when instructed by the teacher to underline or highlight key words and phrases in the description, and working with a partner. E.g., "Sara walks from her home to the store. Halfway to the store, she realizes that she forgot to bring money, so she turns around, returns home, gets her money, and then walks all the way to the store. Graph time on the horizontal axis and distance from home on the vertical axis."	Interpret a linguistically complex mathematical description in order to represent the relationship in a graph (or choose the best answer from a group of graphs), while working with a partner. E.g., "Sara walks from her home to the store. Halfway to the store, she realizes that she forgot to bring money, so she turns around, returns home, gets her money, and then walks all the way to the store. Graph time on the horizontal axis and distance from home on the vertical axis."	

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Reading Continued	so she turns around [gloss with a sketch of a stick figure facing home], returns [gloss: goes back] home, gets her money, and then walks all the way to the store [gloss with a sketch of a stick figure at the store]. Graph time on the horizontal axis and distance [gloss: how far] from home on the vertical axis."	she realizes [gloss: remembers] that she forgot to bring money, so she turns around [gloss with a sketch of a stick figure facing home], returns [gloss: goes back] home, gets her money, and then walks all the way to the store [gloss with a sketch of a stick figure at the store]. Graph time on the horizontal axis and distance [gloss: how far] from home on the vertical axis."	Graph time on the horizontal axis and distance from home on the vertical axis."			

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students explain the relationship between the independent and dependent variables of a function from one of multiple representations (i.e., table, graph, functional notation) for a given context. Describing functional relationships could include details such as identifying the independent and dependent variables, noting what happens to one variable as the other increases, and evaluating the functions for given values. The example described in the writing strand below represents an activity that might occur at the beginning of a unit because it can be concretely represented, which supports both mathematical understanding and communication. For example, students at lower English language proficiency levels will point to and label the concrete representation to demonstrate their understanding of the content.

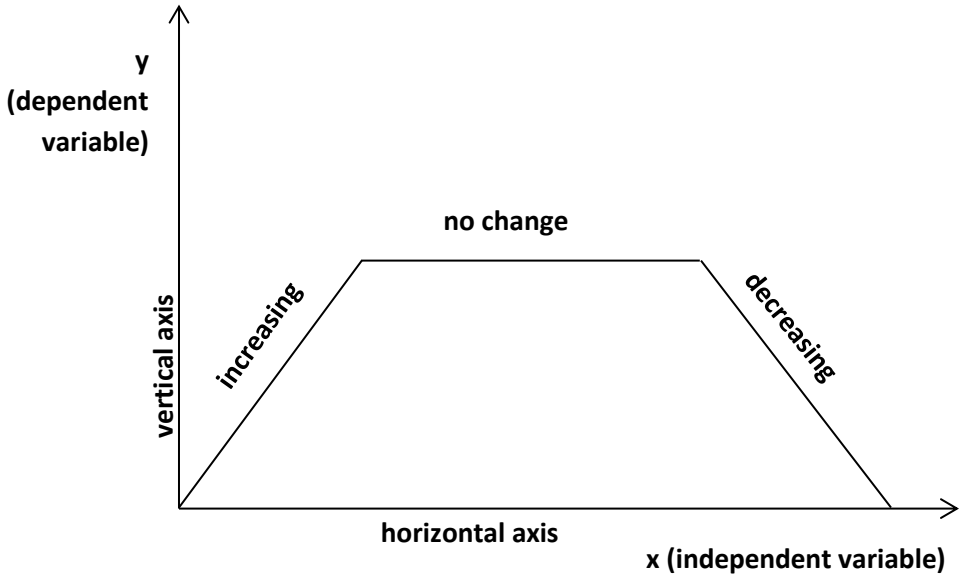
As mathematical ideas develop across a unit of study, teachers can create a poster sized unit anchor chart that summarizes important mathematical ideas (e.g., independent and dependent variables, increasing/decreasing, and using representations as tools for understanding and communicating). The teacher should facilitate recording this collective class knowledge and making the chart accessible for all students throughout the unit. Students should also maintain a personal version of these ideas that they can reference independently. The linguistic complexity can be high for tasks like this because of both mathematical words and contextual words like paperclips, paperclip chain, and length.

COGNITIVE FUNCTION: Students at all levels of English language proficiency indentify the independent and dependent values in a function and use them to DESCRIBE the relationship between them.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Writing	Describe in words, phrases, or simple sentences, the relationship between the independent and dependent values of a function in a real-life context using an illustrated word bank with contextual words, sentence frames, an "if-then" graphic organizer, and a unit anchor chart, while working with a partner. The independent variable is _____(e.g., paper clips). The dependent variable is _____(e.g., length). [labels on the "if-then" graphic organizer] If ____(#)_____(variable), then ____(#)_____(variable).	Describe in simple sentences the relationship between the independent and dependent values of a function in a real- life context using an illustrated word bank for contextual words, sentence frames, and a unit anchor chart, while working with a partner. The independent variable is _____(e.g., paper clips). The dependent variable is _____(e.g., length). If ____(#)_____(variable), then ____(#)_____(variable).	Describe in complete sentences the relationship between the independent and dependent variable of a function in a real- life context using an illustrated word bank for contextual words, unit anchor chart, and working with a partner. E.g.,"The independent variable is paper clips, and the dependent variable is the length of the chain. Eight paper clips make a ten inch chain."	Describe in compound and/or complex sentences, the relationship between the independent and dependent values of a function in a real-life context using a suggested word list (e.g., if, then, independent variable, dependent variable), a unit anchor chart, and working with a partner. E.g., "The independent variable is paper clips, and the dependent variable is the length of the chain. If there are eight paper clips in the chain, then the length of the chain will be ten inches ."	Describe in compound or complex sentences the relationship between the independent and dependent values of a function in a real-life context using a required word list (e.g., if, then, independent variable, dependent variable) working with a partner. E.g., "The independent variable is paper clips, and the dependent variable is the length of the chain. As the number of paper clips increases, so does the length of the chain. If there are eight paper clips in the chain, then the length of the chain will be ten inches ."	

Unit 1 Anchor Chart: Representations of a Function

Graph



Table

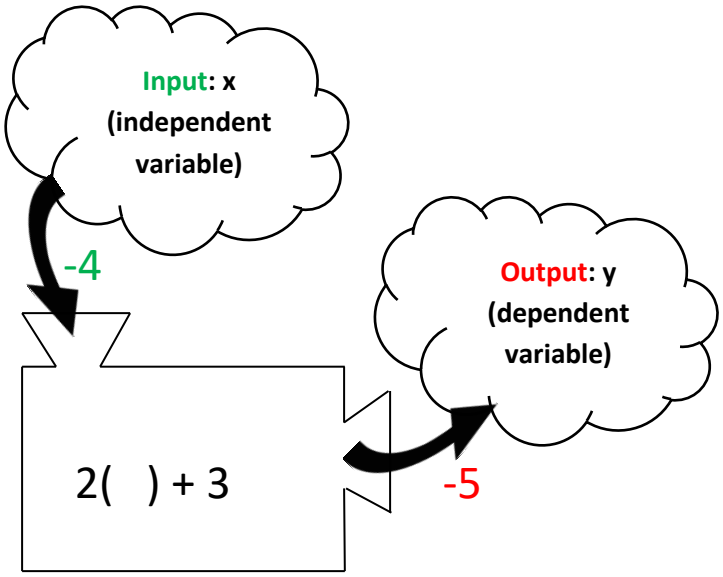
Input x (independent variable)	Output y (dependent variable)

Equation

$f(x) = 2x + 3$

$f(-4) = 2(-4)+3 = -5$

input output



This collection of images can be used to “fully illustrate the problem.” Teachers or peers reading the story could point and gesture to these images helping listeners make sense of unfamiliar contextual words. The images are shown here next to the text that would be read aloud. Since this is a listening task, teachers should <u>not</u> print the story with the images and give it to students; this would change the task to reading.

Kendra is speeding 

(Gesture to several numbers on the speedometer greater than 70.)

along the highway 

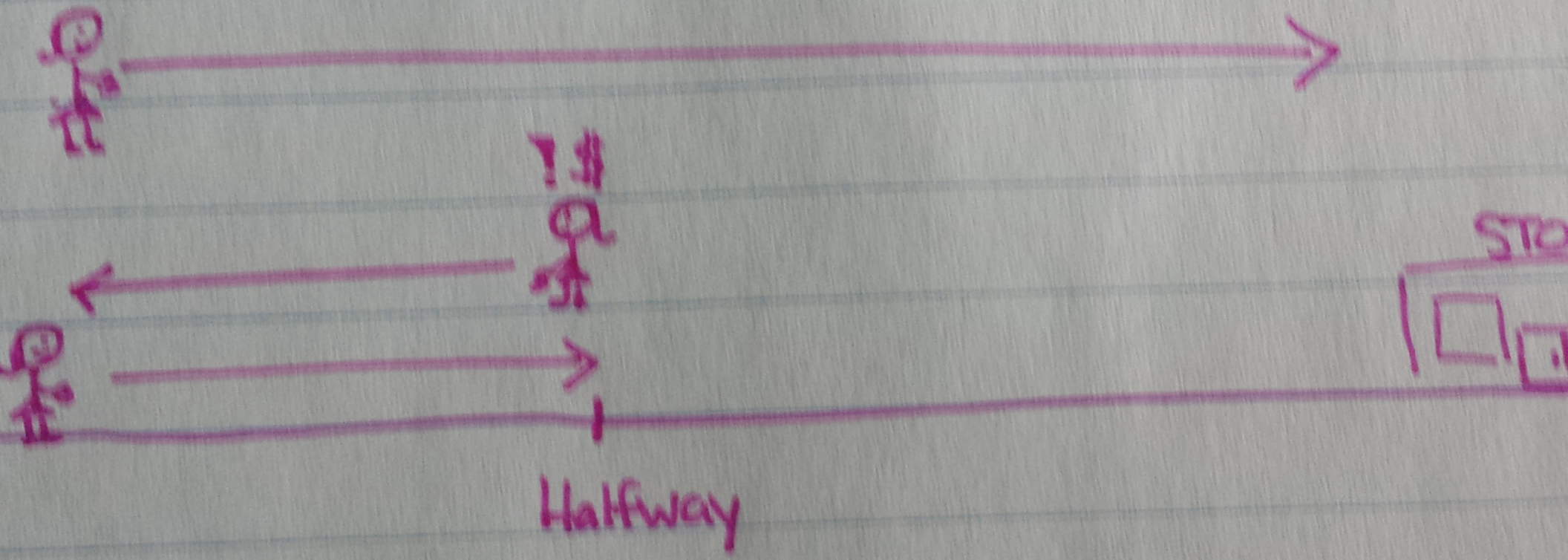
and is stopped by a police officer. 

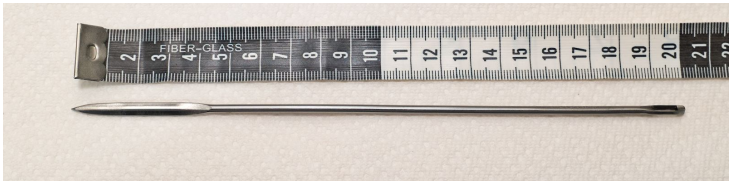
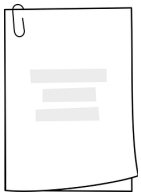
The officer gives her a ticket 

and then she continues on her way. 

This is an example of an illustrated reference sheet, with images quickly found on the internet, that students can use to make sense of unfamiliar contextual words while they are listening, and then, using what they heard to represent the story mathematically. In this case the illustrated reference sheet also contains short and intentionally simplistic definitions of the words to facilitate enduring sense making and connections to the pictures. This reference sheet supports students like the fully illustrated story for students at levels 1 and 2, but increases the language demand as students find and reference the words as needed rather than having the speaker point and gesture to the images.

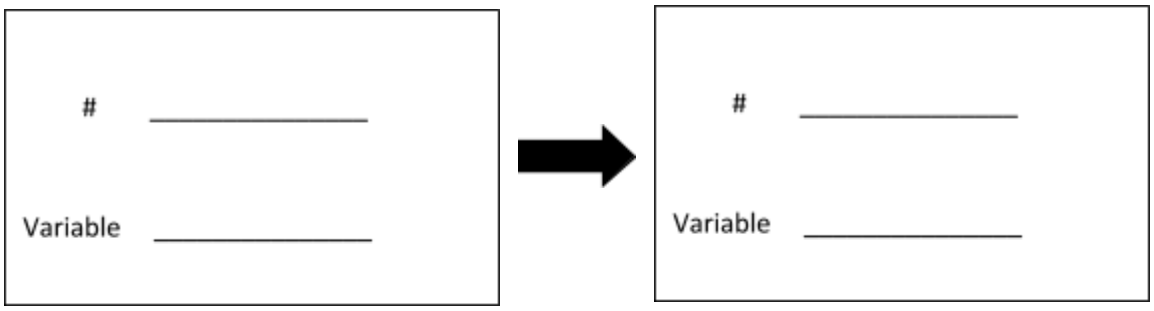
speeding Speeding is driving above the speed limit.	
highway A highway is a road.	
police officer A police officer enforces the law.	
ticket A police officer gives tickets to people who do not follow the rules.	


!$
STO
Halfway

Length - Length is a measurement from end to end.	 "The **length** of this needle is 20 centimeters."
Paper clip A paper clip holds paper together.	 
Paper clip chain A paper clip chain is paper clips connected together.	
Number #	3 47 1,300,421 0.13 3/4

IF

THEN



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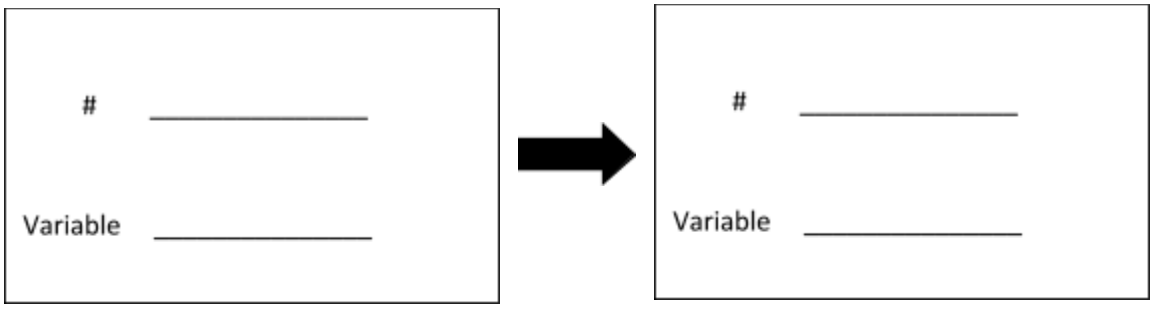

IF

THEN