Unit 2 - Representing and Solving Linear Functions
Algebra I

Friday, December 11, 2015, 11:20AM

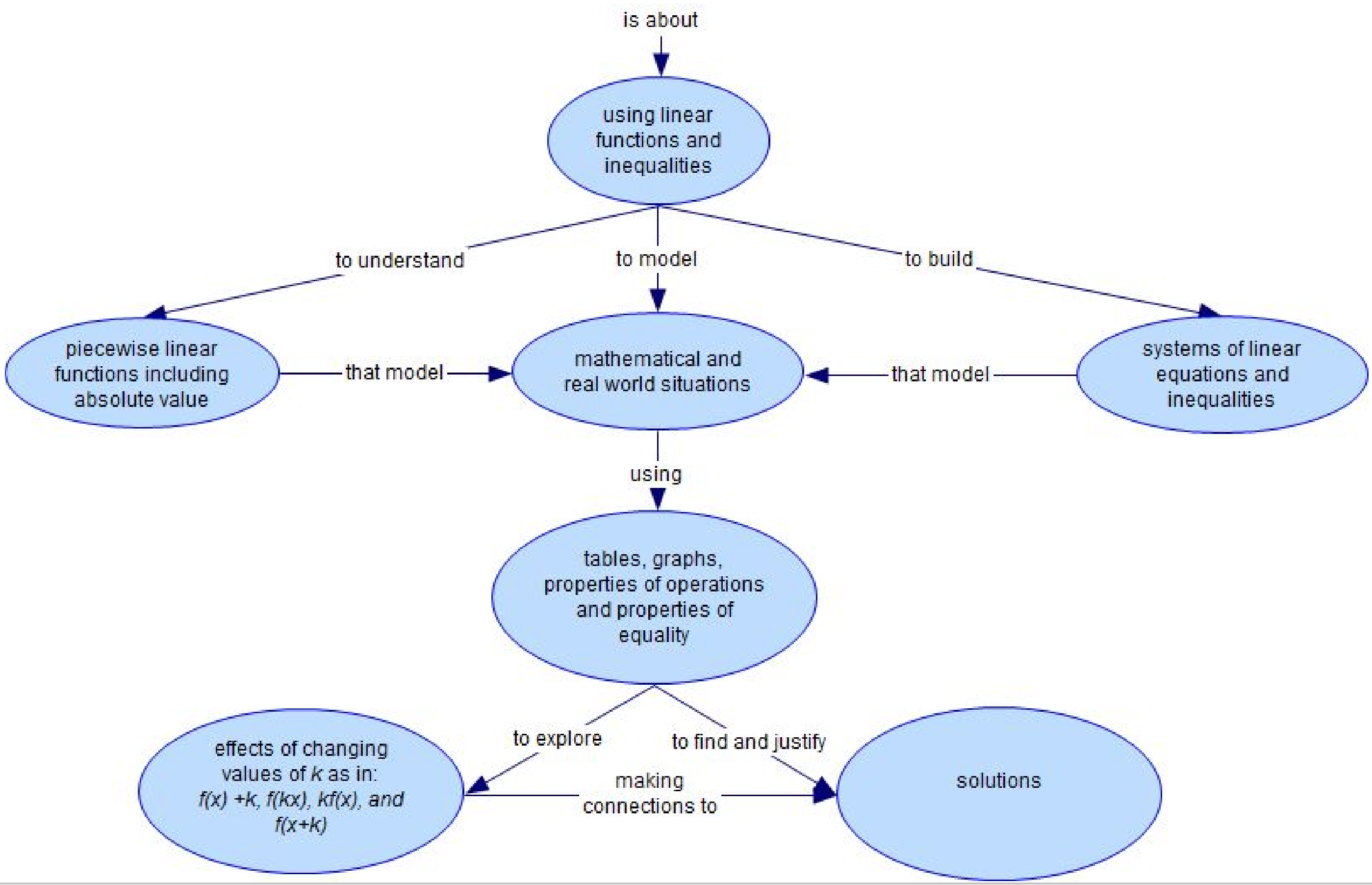
Common Core > 2015-2016 > Grade 9 > Mathematics > Algebra I (**CC**) > Week 5 - Week 12

Common Core Initiative

Overarching Questions and Enduring Understandings

In what ways can equations and inequalities and systems of equations and inequalities model constraints and relationships between quantities? What processes and representations can be used to find and interpret solutions?

Graphic Organizer



Unit Abstract

In eighth grade, students studied how linear functions represented by equations, tables, and graphs can be used to model and solve real world situations. Particular attention was given to using the rate of change and initial value from multiple representations to model the relationship between two values with the generalizable function, y=mx+b. In Algebra 1, students continue using and making connections between representations of linear functions. A common misconception when making connections between representations is that constant rate of change and slope can be used interchangeably when "a linear ***function*** does not have slope, but the ***graph*** of a non-vertical line has a slope." (*High School Functions* progression document, page 6) Unit 2 establishes a deep understanding of the characteristics of linear functions. This understanding includes exploring linear functions geometrically by analyzing the effects of transformations on the graph by replacing f(x) by f(x) + k, kf(x), f(kx) and f(x + k) for specific values of k. These understandings allow students to compare linearity to function families studied in future units (e.g., exponential, quadratic).

In this unit, students will extend their focus from slope intercept form to reasoning about standard form and point slope form. Students begin to make distinctions about which of these forms are most beneficial when modeling a real world situation. Different contexts lend themselves to different forms of linear equations. Students may build a function to model a situation, using parameters from that situation (e.g., rate of change, start value, ordered pair). Other situations are more efficiently modeled with standard form (e.g., Dana purchased 3 brauts and 4 drinks for $8.50). Symbolic manipulation from one form to another can reveal new characteristics of the function or assist in solving systems of equations.

In 6th and 7th grade students solved one and two step equations and inequalities algebraically. In 8th grade students solved linear equations using graphs, tables, and algebraic manipulation. In this unit, students will apply what they know about solving equations and inequalities to solving multi-step inequalities which include variables on both sides. Students will make sense of what a solution means for an equation compared to an inequality. Students will extend their understanding of solving linear equations with two variables. First, they will manipulate equations to solve for specific variables. Second, students will justify their reasoning by supplying mathematical properties to explain each step in solving an equation. This work will help set the groundwork for mathematical proofs in tenth grade.
In addition, students will extend their 8th grade understanding of solving systems of linear equations to include systems of linear inequalities as representations of real world situations. Students will solve systems of linear equations exactly (e.g. with substitution principle, combination/elimination), and approximately (e.g., with graphs) with a new emphasis on the conceptual understanding and justification of why these strategies work. Students will compare and contrast the benefit of using each of these strategies in different situations.
In this unit students are using what they know about linear functions to build new understandings of piecewise linear functions including absolute value functions. In addition, students will use tables and graphs to solve absolute value equations as described in HSA.REI.D.11. The Michigan State Standards no longer require students to be able to algebraically solve piecewise and absolute value functions.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Algebra

Seeing Structure in Expressions

HSA-SSE.B. Write expressions in equivalent forms to solve problems.

- HSA-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

- HSA-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.HSA-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

For example, rearrange Ohm's law V = IR to highlight resistance R.

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the CCSS-M standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations & Inequalities

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the

Reasoning with Equations & Inequalities

HSA-REI.A. Understand solving equations as a process of reasoning and explain the reasoning.

- HSA-REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

HSA-REI.B. Solve equations and inequalities in one variable.

- HSA-REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

HSA-REI.C. Solve systems of equations.

- HSA-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- HSA-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

High School: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Linear, Quadratic, and Exponential Models

HSF-LE.A. Construct and compare linear and exponential models and solve problems.

  - HSF-LE.A.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpreting Functions

HSF-IF.B. Interpret functions that arise in applications in terms of the context.

- HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★
- **HSF-IF.C. Analyze functions using different representations.**
  - HSF-IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - HSF.IF.C.7.b. Graph square root, cube root,and piecewise-defined functions, including step functions and absolute value functions.
- HSF-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models
HSF-LE.A. Construct and compare linear and exponential models and solve problems.

- HSF-LE.A.1. Distinguish between situations that can be modeled with linear functions and with [nonlinear functions] exponential functions.
  - HSF-LE.A.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- HSF-LE.A.2. Construct linear and exponential

	functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **HSF-LE.B. Interpret expressions for functions in terms of the situation they model.** • HSF-LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context. **<u>Developed and/or utilized across multiple units</u>** **Quantities** **HSN-Q.A. Reason quantitatively and use units to solve problems.** • HSN-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling. • HSN-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Creating Equations** **HSA-CED.A. Create equations that describe numbers or relationships.** • HSA-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **Reasoning with Equations and Inequalities** **HSA-REI.D. Represent and solve equations and inequalities graphically.** • HSA-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **Interpreting Functions** **HSF-IF.A. Understand the concept of a function and use function notation.** • HSF-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). **HSF-IF.B. Interpret functions that arise in applications in terms of the context.** • HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

	• HSF-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ • HSF-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **HSF-IF.C. Analyze functions using different representations.** • HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Building Functions** **HSF-BF.A. Build a function that models a relationship between two quantities.** • HSF-BF.A.1. Write a function that describes a relationship between two quantities. o HSF-BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
Essential/Focus Questions 1. How do changes in values of the parameters in a linear function change the behavior of its graph and/or table? 2. What are some possible strategies for solving systems of equations and inequalities? When might one strategy be more efficient than another? 3. What does it mean to find solutions for equations, inequalities, and systems of equations and inequalities? How are the processes similar and different? 4. How can understanding linear functions assist in investigating and representing piecewise linear functions?	**Key Concepts** rate of change of a linear function (slope of the graph, common difference) linear patterns (recursive and explicit) models of linear functions (tables, graphs, and equations) intercepts parallel and perpendicular lines piecewise linear functions (absolute value, step) properties of operations (associative, commutative, distributive) properties of equality (addition/subtraction, multiplication/division) solutions (to equations, inequalities, systems of equations, and systems of inequalities) solution processes (algebraic, graphical, numeric) equivalent linear expressions transformations of a linear function
Assessment Tasks Formative Assessment Overview 1 Student Handout 1 Independent Practice 1 Formative Assessment Overview 2	**Intellectual Processes** **Standards for Mathematical Practice** ***Students will have opportunities to:*** • **construct viable arguments and critique the reasoning of others** when explaining each step in solving a linear equation or inequality; • **reason abstractly and quantitatively** when decontextualizing a situation in order to write a linear equation, inequality or system of linear equations or inequalities; and • **model with mathematics** by creating and using

	multiple representations to organize, record, and communicate information about relationships between variables.
Lesson Sequence Lesson 1 Overview Student Handout Professional Learning Task-Teacher Reflection Video of the Lesson Lesson 2 Overview	**Resources** Unit Resources