ELD STANDARD 3: The Language of Mathematics	MAISA Algebra I, Unit 3: Exponential Functions

CONNECTIONS: Michigan Academic Standards for Mathematics - Algebra I

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students, given an equation of an exponential "parent" function y = a(b)^(x-h)+k, will listen to a description of two changes that will be made to the parameters (a, b, h, or k), select from several possible choices the graph they believe will show the key attributes, and then verify their answer by completing a table and graph (use of a graphing calculator optional).

One strategy for creating six answer choices is to vary how the changes in parameters affect the graph. For example, if changing x to (x+h) and a to (-a), show answer choices (a) graph shifted to the right, (b) flipped over x-axis and shifted right, (c) flipped over x-axis and shifted left, (d) shifted right, (e) flipped over y-axis and shifted right, (f) flipped over y-axis and shifted left

Note: The current task gives students no opportunity to reflect on what they did right **or** what they did wrong. Students will learn more with reflection than without. Teachers can incorporate a writing or speaking element where students investigate why their prediction was right or wrong, either in partner work or whole class discussion.

COGNITIVE FUNCTION: Students at all levels of language proficiency **HYPOTHESIZE** and **RECOGNIZE** how changes in values of the parameters in an exponential function affect the key attributes of the graph (i.e., y-intercept, increasing/decreasing, horizontal/vertical shift).

	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Listening	Listen to a description read aloud multiple times (with purposeful pauses and points to the part of the paper they should be working on) of two changes made to the equation of an exponential parent function, select from several possible choices the graph that will show the key attributes (i.e., y-intercept, increasing/decreasing, horizontal/vertical shift) and then verify the answer by completing a table and a graph (use of a graphing calculator optional), referring to a related completed model (example in completed model sheet), working with a partner at a higher level of English language proficiency. E.g., "[The teacher writes the parent function and points to it as a reference throughout the task.] In the equation [pause and point], y equals 2 [pause and point] times 3 [pause and point] to the x power [pause and point],	Listen to a description read aloud multiple times (with purposeful pauses and points to the part of the paper they should be working on) of two changes made to the equation of an exponential parent function, select from several possible choices the graph that will show the key attributes (i.e., y-intercept, increasing/decreasing, horizontal/vertical shift) and then verify the answer by completing a table and a graph (use of a graphing calculator optional), referring to a related completed model, working with a partner at a higher level of English language proficiency. E.g., "[point to a blank place on the paper for the student to write on] In the equation [pause] y equals 2 [pause] times 3 [pause] to the x power [pause], the two is changed to negative two [pause], and 3 is added to the exponent x [pause], making the new equation [pause] y	Listen to a description read aloud multiple times (with purposeful pauses) of two changes made to the equation of an exponential parent function, select from several possible choices the graph that will show the key attributes (i.e., y-intercept, increasing/decreasing, horizontal/vertical shift) and then verify the answer by completing a table and a graph (use of a graphing calculator optional), working with a partner at a higher level of English language proficiency. E.g., "In the equation [pause] y equals 2 [pause] times 3 [pause] to the x power [pause], the two is changed to negative two [pause], and 3 is added to the exponent x [pause], making the new equation [pause] y equals negative two [pause] times 3 [pause] to the x plus three power [pause].	Listen to a description read aloud (with purposeful pauses) of two changes made to the equation of an exponential parent function, select from several possible choices the graph that will show the key attributes (i.e., y-intercept, increasing/decreasing, horizontal/vertical shift) and then verify the answer by completing a table and a graph (use of a graphing calculator optional), working with a partner at a lower level of English language proficiency. E.g., "In the equation [pause] y equals 2 [pause] times 3 [pause] to the x power [pause],	Listen to a description read aloud (with purposeful pauses) of two changes made to the equation of an exponential parent function, select from several possible choices the graph that will show the key attributes (y-intercept, increasing/decreasing, horizontal/vertical shift) and then verify the answer by completing a table and a graph (use of a graphing calculator optional), working with a partner at a lower level of English language proficiency.	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening Continued	Change the two to negative two [pause and point], and add 3 to the exponent x [pause and point]. This makes the new equation [pause and point to a blank place on the paper for the student to write on] y equals negative two [pause and point] times 3 [pause and point] to the x plus three power [pause and point]. [point to the answer choices provided on the paper.] Choose the graph that you believe will best represent the new equation [pause]. Then make a table [pause, point to the place on the student's paper where a blank table is provided] and graph [pause, point to the place on the student's paper where a blank graph is provided] of at least 4 ordered pairs [pause, hold up 4 fingers] to verify your choice to make sure it is correct." *sample activity sheet and completed model provided in supports	equals negative two [pause] times 3 [pause] to the x plus three power [pause]. [point to the answer choices provided on the student's paper] Choose the graph that you believe will best represent the new equation [pause]. Then make a table [pause, point to the place on the student's paper where a blank table is provided] and graph [pause, point to the place on the student' paper where a blank graph is provided] of at least 4 ordered pairs [pause, hold up 4 fingers] to verify your choice to make sure it is correct." *sample activity sheet and completed model provided in supports	Choose the graph that you believe will best represent the new equation [pause]. Then make a table [pause] and graph [pause] of at least 4 ordered pairs [pause] to verify your choice to make sure it is correct." *sample activity sheet provided in supports	the two is changed to negative two [pause], and 3 is added to the exponent x [pause], making the new equation [pause] y equals negative two [pause] times 3 [pause] to the x plus three power [pause]. Choose the graph that you believe will best represent the new equation [pause]. Then make a table [pause] and graph [pause] of at least 4 ordered pairs [pause] to verify your choice to make sure it is correct." *sample activity sheet provided in supports	E.g.: "In the equation [pause] y equals 2 [pause] times 3 [pause] to the x power [pause], the two is changed to negative two [pause], and 3 is added to the exponent x [pause], making the new equation [pause] y equals negative two [pause] times 3 [pause] to the x plus three power [pause]. Choose the graph that you believe will best represent the new equation [pause]. Then make a table [pause] and graph [pause] of at least 4 ordered pairs [pause] to verify your choice to make sure it is correct." *sample activity sheet provided in supports	

ELD STANDARD 3: The Language of Mathematics				MAISA Unit Algebra I, Unit 3 Exponential Functions		

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students explain how the rules of exponents can be used to simplify expressions. The example below is written for multiplying expressions with the same base. Similar supports could be generated for other exponent rules. It is more mathematically precise to say "x used as a factor" vs "x multiplied by itself" however, you may expect students to respond using either phrase.

Note: The teacher and students should work on an example together, creating "sentence models" (sentences that will be spoken, with underlined terms replaced as appropriate) on the board or on reference sheets used by student groups. The sentence model should not be reproduced for students to simply "fill in the blank." Rather students should use the sentence models as scaffolding, helping them recreate the sentences in a different context.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **JUSTIFY** one of the rules of exponents.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking	Justify simplifying an exponential expression with multiple simple sentences using sentence models (where underlined examples would be replaced with specific problems) and pointing to their own symbolic representation instead of reading the expression aloud while working in a small group with mixed abilities, mathematically and linguistically, where students with higher levels of English language proficiency model appropriate responses. [Given a problem similar to x^2 * x^3 = x^5, the student would substitute appropriate expressions from the new problem in place of underlined expressions in the model] <u>x^2</u> uses <u>2</u> factors of x so <u>x^2 = x*x.</u> <u>x^3</u> uses <u>3</u> factors of x so <u>x^3 = x*x*x.</u>	Justify simplifying an exponential expression with multiple simple sentences using sentence models (where underlined examples would be replaced with specific problems) and sentence stem(s) with choices while working in a small group with mixed abilities, mathematically and linguistically, where students with higher levels of English language proficiency model appropriate responses. [Given a problem similar to x^2 * x^3 = x^5, the student would substitute appropriate expressions from the new problem in place of underlined expressions in the model] <u>x^2</u> uses <u>2</u> factors of x so <u>x^2 = x*x</u>. <u>x^3</u> uses <u>3</u> factors of x so <u>x^3 = x*x*x</u>.	Justify simplifying an exponential expression with multiple sentences using sentence models (where highlighted example would be replaced with a specific problem) and sentence stem(s) and a suggested word list(e.g., equivalent, exponent(s), base(s), sum/adding, powers) while working in a small group with mixed abilities, mathematically and linguistically, where students with higher levels of English language proficiency model appropriate responses. [Given a problem similar to x^2 * x^3 = x^5, the student would substitute appropriate expressions from the new problem in place of underlined expressions in the model] x^2 uses 2 factors of x so x^2 = x*x.	Justify simplifying an exponential expression using compound and/or complex sentences and a suggested word list (e.g., equivalent, exponent(s), base(s), sum/adding, factor) while working in a small group with mixed abilities, mathematically and linguistically while working in a small group with mixed abilities, mathematically and linguistically, where students with higher levels of English language proficiency model appropriate responses. [Given x^2 * x^3 = x^5] E.g., "x^2 uses two factors of x, so it is equivalent to x*x. x^3 uses three factors of x, so it is equivalent to x*x*x. The product will use 2 + 3 = 5 factors of x. This will make x*x*x*x*x which equals x^5.	Justify simplifying an exponential expression using compound and/or complex sentences and a suggested word list (e.g., equivalent, exponent(s), base(s), sum/adding, factor) while working in a small group with mixed abilities, mathematically and linguistically, where students with higher levels of English language proficiency model appropriate responses. [Given x^2 * x^3 = x^5] E.g., "The exponents represent how many times x is used as a factor. x^2 uses two factors of x, so it is equivalent to x*x. x^3 uses three factors of x, so it is equivalent to x*x*x. The product will use 2 + 3 = 5 factors of x. This will make x*x*x*x*x which equals x^5.	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking Continued	x^2 * x^3 uses 2 + 3 = 5 factors of x. So, x^2 * x^3 = (x*x)*(x*x*x) = x^(2+3) = x^5. E.g., "This [pointing to expression like x^2] is two factors. It equals this [point to expression like x*x]. This [point to expression like x^3] had three factors. It equals this [point to expression like x*x*x]. This [point to expression like x^2 *x^3] is 2 + 3 factors. It equals this [point to expression like x*x*x*x*x] and [point to expression like x^5]. Same bases [pointing to both x's in original product] means add powers [point to 2 + 3]."	x^2 * x^3 uses 2 + 3 = 5 factors of x. So, x^2 * x^3 = (x*x)*(x*x*x) = x^(2+3) = x^5. When multiplying powers with the same bases, __________ (add/multiply) the ___________ (bases/exponents).	x^3 uses 3 factors of x so x^3 = x*x*x. x^2 * x^3 uses 2 + 3 = 5 factors of x. So, x^2 * x^3 = (x*x)*(x*x*x) = x^(2+3) = x^5. When multiplying powers with like bases…	The exponents represent how many times x is used as a factor. The end result is x used as a factor 5 times, which is equivalent to the sum of the exponents because they have like bases. So the product of two terms with like bases is to add the powers without changing the base."	The end result is x used as a factor 5 times, which is equivalent to the sum of the exponents because they have like bases. So the product of two terms with like bases is to add the powers without changing the base."	

ELD STANDARD 3: The Language of Mathematics	MAISA Unit Algebra I, Unit 3: Exponential Functions

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will read a real-world situation and will be able to analyze the exponential function described in the scenario to create a formula, table, and graph of the function. This formative task would overlap with a writing task due to the task requiring reading comprehension of the scenario as well as creating in writing different representations of the exponential function described in the scenarios. The example below is taken from regentsprep.org (see Regents Prep Task Sheet in this unit) which also includes an answer key. It is recommended to start with Question #1 (chess scenario) with all students as it is the most concrete example and easiest for students to imagine. Other similar contexts are available at the same site.

Recommended text resources (all available on Amazon.com):
-The Rajah's Rice: A Mathematical Folktale from India (Tales of Myth & Legend) (ISBN-10: 0716765683 ISBN-13: 978-0716765684)
-The King's Chessboard (ISBN-10: 0140548807 ISBN-13: 978-0140548808)
-One Grain Of Rice: A Mathematical Folktale (ISBN-10: 059093998X ISBN-13: 978-0590939980)

COGNITIVE FUNCTION: Students at all levels of English language proficiency will **EVALUATE** the written scenarios and **CREATE** three representations (formula, table, graph) of the exponential function described in the scenarios.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Analyze a fully illustrated version of linguistically complex mathematical story in order to model the story describing a real-world exponential function using an illustrated word bank with both mathematical and contextual words while working with a partner. (A fully illustrated version of the story is provided in the supports.)	Analyze a glossed version of linguistically complex mathematical story in order to model the story describing a real-world exponential function using an illustrated word bank with both mathematical and contextual words while working with a partner. E.g., There is a well-known [gloss: famous] fable [gloss: story] about a man from India who invented [gloss: made] the game of chess, as a gift for his king. The king was so pleased [gloss: happy]with the game that he offered to grant [gloss: give] the man any request within reason [gloss: anything he asked for].	Analyze a glossed version of linguistically complex mathematical story in order to model the story describing a real-world exponential function while working with a partner and using an illustrated word bank with both mathematical and contextual words. E.g., There is a well-known [gloss: famous] fable [gloss: story] about a man from India who invented [gloss: made] the game of chess, as a gift for his king. The king was so pleased [gloss: happy]with the game that he offered to grant [gloss: give] the man any request within reason [gloss: anything he asked for].	Analyze a linguistically complex mathematical story in order to model the story describing a real-world exponential function while working with a partner and using an illustrated word bank with both mathematical and contextual words. E.g., There is a well-known fable about a man from India who invented the game of chess, as a gift for his king. The king was so pleased with the game that he offered to grant the man any request within reason.	Analyze a linguistically complex mathematical story in order to model the story describing a real-world exponential function while working with a partner and using an illustrated word bank with both mathematical and contextual words. E.g., There is a well-known fable about a man from India who invented the game of chess, as a gift for his king. The king was so pleased with the game	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading Continued		The man asked for one grain [gloss: piece] of wheat to be placed on the first square of the chess board, two grains to be placed on the second square, four on the third, eight on the fourth, etc., doubling the number of grains of wheat each time, until all 64 squares on the board had been used. The king, thinking this to be a small request, agreed. A chess board has 64 squares. How many grains of wheat did the king have to place on the 64th square of the chess board?	The man asked for one grain [gloss: piece] of wheat to be placed on the first square of the chess board, two grains to be placed on the second square, four on the third, eight on the fourth, etc., doubling the number of grains of wheat each time, until all 64 squares on the board had been used. The king, thinking this to be a small request, [gloss: wish] agreed. A chess board has 64 squares. How many grains of wheat did the king have to place on the 64th square of the chess board?	The man asked for one grain of wheat to be placed on the first square of the chess board, two grains to be placed on the second square, four on the third, eight on the fourth, etc., doubling the number of grains of wheat each time, until all 64 squares on the board had been used. The king, thinking this to be a small request, agreed. A chess board has 64 squares. How many grains of wheat did the king have to place on the 64th square of the chess board?	that he offered to grant the man any request within reason. The man asked for one grain of wheat to be placed on the first square of the chess board, two grains to be placed on the second square, four on the third, eight on the fourth, etc., doubling the number of grains of wheat each time, until all 64 squares on the board had been used. The king, thinking this to be a small request, agreed. A chess board has 64 squares. How many grains of wheat did the king have to place on the 64th square of the chess board?	

EXAMPLE CONTEXT FOR LANGUAGE USAGE: The strand below is designed to be used as a formative assessment task later in the unit when students should understand that exponential functions with a base greater than one will eventually be greater than linear functions. When the base is very close to 1, however, the exponential function will look like a linear function for small values of x: (1+a)^x is approximately equal to 1+ax for small values of x, so the graph looks almost horizontal, which requires students to draw upon reasoning developed earlier in the unit.

In the example below, students compare a linear and an exponential function. However, similar supports could be used when students are asked to compare two linear, two exponentials, or in later units, linear and other nonlinear functions. Solution 1 (shown at the Illustrative Mathematics link below) is used for the sample responses shown below. However, teachers should support students to also understand Solution 2 through their work or class discussion. Solution 1 demonstrates inductive reasoning which can be misleading in mathematics, because making a generalization based on several cases may still not represent all cases. Solution 2 demonstrates deductive reasoning which is central to mathematics to show a thing is true for all cases.

Students should be encouraged to use technology to assist their thinking using multiple representations. A table, graph, and symbolic representation are different because a table shows only a few data points, a graph shows infinite data points (within a finite interval), and an equation represents all data points. The example in the strand below represents a formative assessment task. However, teachers should use the performance indicators with similar tasks throughout the unit.

COGNITIVE FUNCTION: Students at all levels of language proficiency compare two functions and **JUSTIFY** their thinking.
The example in the strand below is taken from: https://www.illustrativemathematics.org/content-standards/HSF/LE/A/3/tasks/368

	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Writing	Write a comparison statement using mathematical symbols and justify reasoning by labeling representations (either given or student generated) of two functions, using a teacher- or student-created illustrated suggested word list (greater than/less than/equal to, linear and/or exponential, value(s), function, table/graph/equation), a sentence frame for the comparison statement, and while working with a partner. Comparison statement: ________ (<,>,=)_______ when x is ______(small/big) Justification: [label a representation of the functions with the words "greater than/less than/equal to" and drawing an arrow to, circling, or otherwise indicating what those words are describing]	Write a comparison statement and justify reasoning by labeling representations (either given or student generated) of two functions, using a teacher- or student-created illustrated suggested word list (greater than/less than/equal to, linear and/or exponential, value(s), function, table/graph/equation), a sentence frame for the comparison statement, and while working with a partner. Comparison statement: ________ is (greater than/less than/equal to)_______ when x is ______(small/big) Justification: [label a representation of the functions with the words "greater than/less than/equal to" and drawing an arrow to, circling, or otherwise indicating what those words are describing]	Write a comparison statement and justify reasoning, in complete sentences, given representations for two functions, a suggested word list (greater than/less than/equal to, linear and/or exponential, value(s), function, table/graph/equation), sentence frames, and while working with a partner. E.g., [Given a table for functions 2x and (1.001)^x] Comparison statement: The _________function is __________(greater than/less than) the ________function when________. (I.e., The exponential function is greater than the linear function when the x-values are big.)	Write a comparison statement and justify reasoning, in complete sentences, using transition words, given representations for two functions, a suggested word list (greater than/less than/equal to, linear and/or exponential, value(s), function, table/graph/equation; transition words such as however, at first, then, eventually), and while working with a partner. E.g., [Given a table for functions 2x and (1.001)^x] Comparison statement:	Write a comparison statement and justify reasoning, in compound and/or complex sentences, using transition words, given representations for two functions, using a required word list (greater than/less than/equal to, linear and/or exponential, value(s), function, table/graph/equation; transition words such as however, at first, then, eventually), and while working with a partner. E.g., [Given a table for functions 2x and (1.001)^x] Comparison statement:	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Writing Continued			Justification: The _______(table/graph/equation) shows __________.	"Eventually the exponential function f(x)=(1.001)^x is greater than the linear function f(x)=2x." Justification: "At first it looks like 2x is growing faster. However, at really large values of x in the table, 1.001^x had larger values than 2x."	"Eventually the exponential function f(x)=(1.001)^x is greater than the linear function f(x)=2x." Justification: "At first it looks like 2x is growing faster. However, when I looked at really large values of x in the table, it showed that 1.001^x had larger values than 2x, once x is large enough."	

Algebra 1 Unit 3 Listening: Activity Sheet

Original Equation: ___________________________________

New Equation: ______________________________________

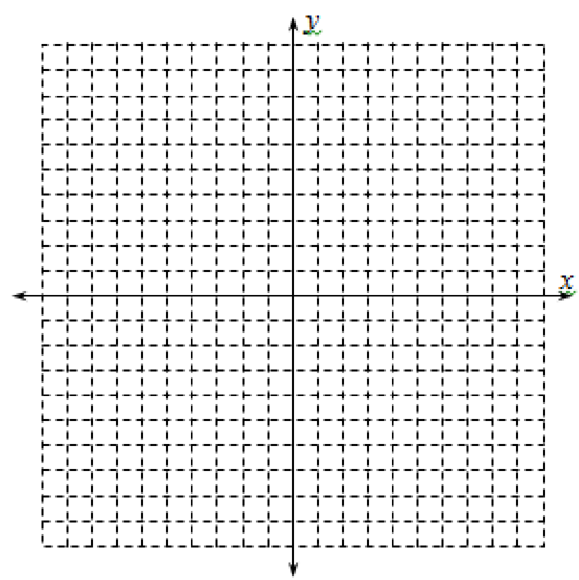
Prediction: (choose an answer)

a.	b.	c.
d.	e.	f.

Table:

x	y

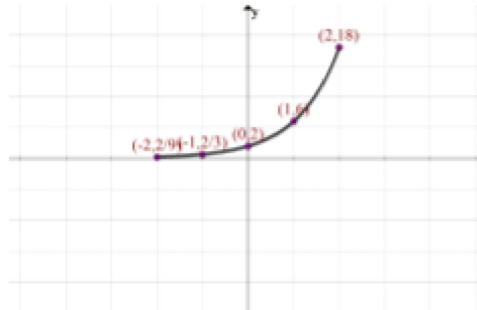
Graph:



Algebra 1 Unit 3 Listening: Activity Sheet

Original Equation: $y=2(3)^x$

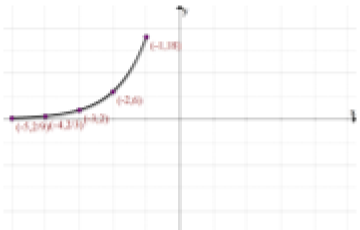
New Equation: $y=-2(3)^{(x+3)}$



x	y
-1	2/3
0	2
1	6
2	18

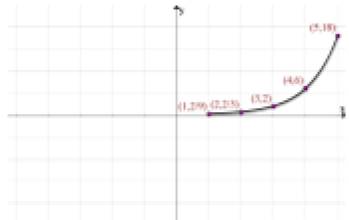
Prediction: (choose an answer; domain limited to 5 length)

a.



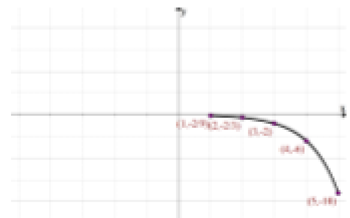
x	y
-4	2/3
-3	2
-2	6
-1	18

b.



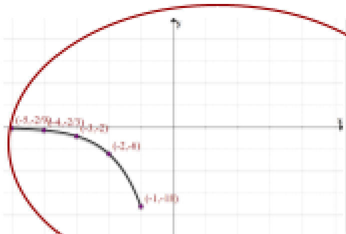
x	y
2	2/3
3	2
4	6
5	18

C.



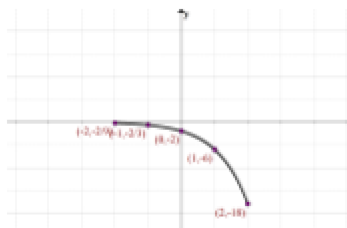
x	y
2	-2/3
3	-2
4	-6
5	-18

d.



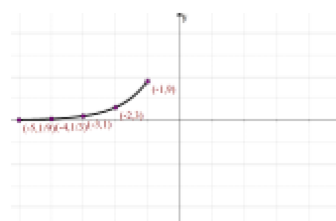
x	y
-4	-2/3
-3	-2
-2	-6
-1	-18

e.



x	y
-1	-2/3
0	-2
1	-6
2	-18

f.



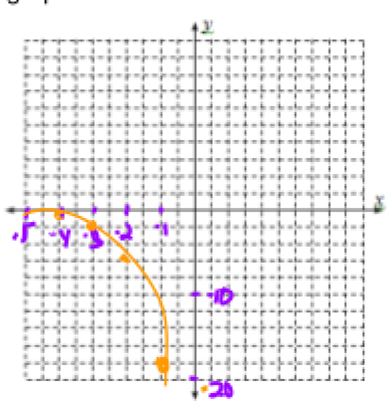
x	y
-4	1/3
-3	1
-2	3
-1	9

$y=-2(3)^{(x+3)}$

table:

x	y
-5	-2/9
-4	-2/3
-3	-2
-2	-6
-1	-18

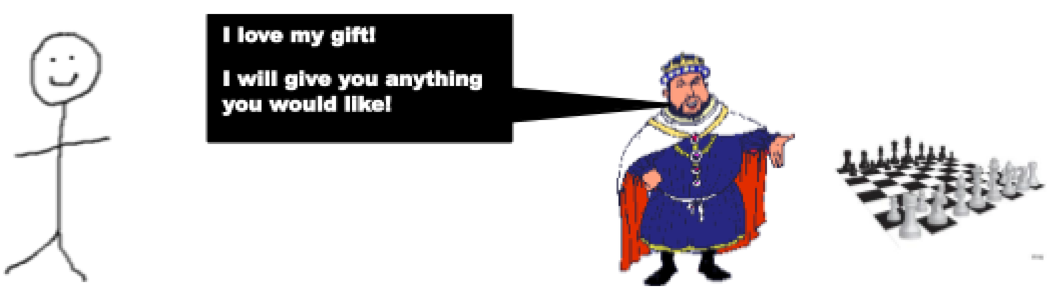
graph:



This is a fable.

A man from India invented a game called chess. The game was a gift for his king.

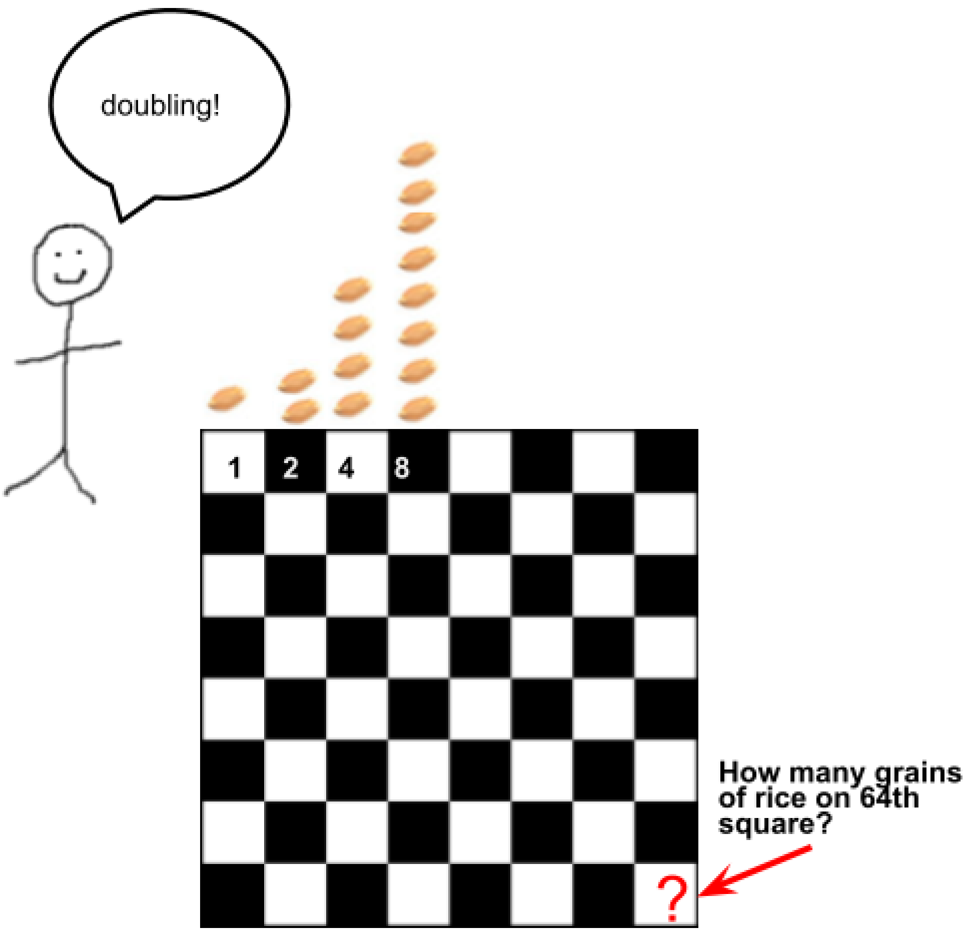
The king was happy. He offered to give the man anything.



All images are from Microsoft Word Office Clip Art unless otherwise noted.

The man said:

- Put one grain of wheat on the first square of the chess board.
- Put two grains on the second square.
- Put four on the third.
- Put eight on the fourth.
- And so on.
- Double the number of grains of wheat each time.
- Use all 64 squares on the chessboard.



A chess board has 64 squares.

How many grains of wheat did the king put on the 64th square of the board?

All images are from Microsoft Word Office Clip Art unless otherwise noted.

Fable: a story that teaches a lesson

India: a country in Asia

from https://tinyurl.com/y6sfktfv

Chess: a game of strategy between two players

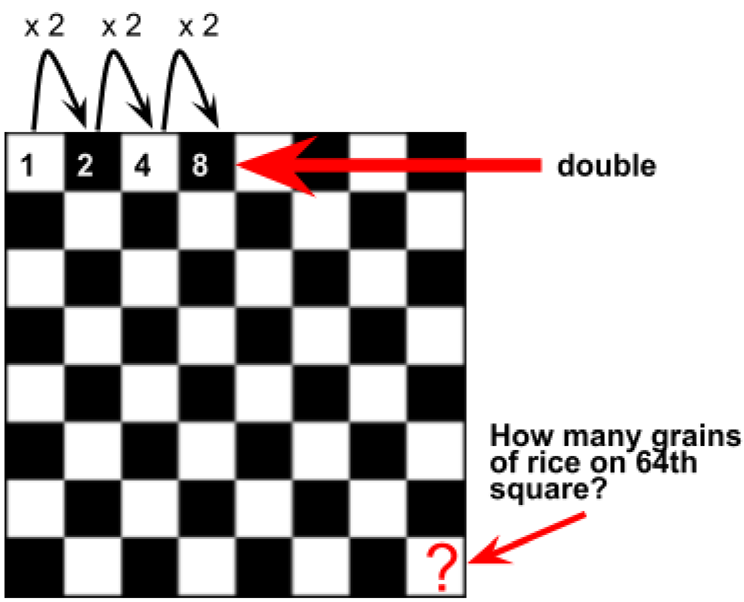
King: male leader of country because of birth or family

Grains of wheat: seeds can grow more wheat. Wheat can be made into flour and used to feed people.

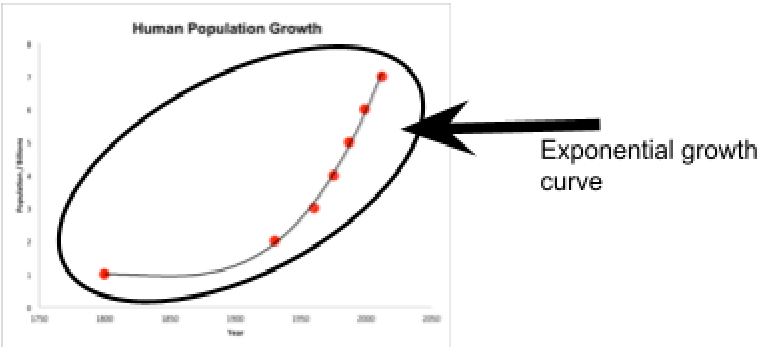
equations show relationships $\_\_ = \_\_$

$2 + 3 = 1 + 4 \quad 64 = 2^6 \quad y = 2x + 1 \quad y = 2^x$

Function: equation

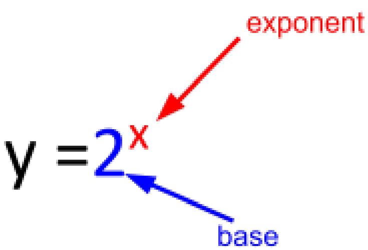


Double: multiply number or quantity by 2.

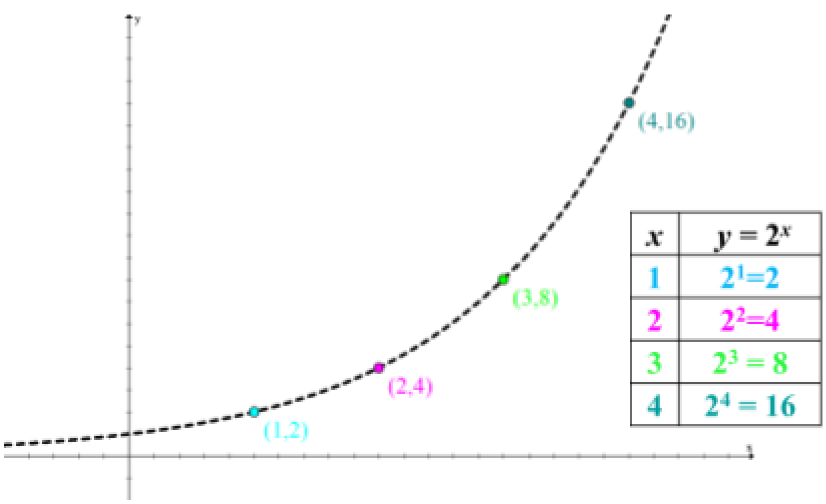


Exponential:

- a function with a number (base) that has a variable exponent $y = 2^x$



- when x increases by 1, the y value is multiplied by a factor (same each time)



Practice with Applied Exponential Growth and Decay

For each of the situations described below, develop a chart, an equation and a graph to illustrate the data. Grab your graphing calculator.

1. **The Fable of the Chess Board and the Grains of Wheat**
There is a well-known fable about a man from India who invented the game of chess, as a gift for his king. The king was so pleased with the game that he offered to grant the man any request within reason. The man asked for one grain of wheat to be placed on the first square of the chess board, two grains to be placed on the second square, four on the third, eight on the fourth, etc., doubling the number of grains of wheat each time, until all 64 squares on the board had been used. The king, thinking this to be a small request, agreed. A chess board has 64 squares. How many grains of wheat did the king have to place on the 64th square of the chess board?
    a. **Complete the chart:**

Number of doubling	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**...**	**20**
Wheat on each square	2															
Pattern	2^1															

    b. **Write a function to illustrate the situation.**

    c. **Plot the data and graph the function for squares 1 through 10.**

2. **Rabbit Population Growth**
In 1995, there were 85 rabbits in Central Park. The population increased by 12% each year. How many rabbits were in Central Park in 2005?
    a. **Complete the chart:**

Year	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
Years since 1995	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of Rabbits (Round to the nearest rabbit.)	95									
Pattern	$85(1.12)^1$									

    b. **Write a function to illustrate the situation.**

    c. **Plot the data and graph the function.**

3. **Bacteria Growth**
A scientist has discovered a new strain of bacteria. The bacteria culture initially contained 1000 bacteria and the bacteria are doubling every half hour.
    a. **Complete the chart:**

Time Interval	**0.5 hr**	**1.0 hr**	**1.5 hr**	**2.0 hr**	**2.5 hr**	**3.0 hr**	**3.5 hr**	**4.0 hr**	**4.5 hr**	**5.0 hr**
30 minute intervals since start	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Bacteria present	2000									
Pattern										

    b. **Write a function to illustrate the situation.**

    c. **Plot the data and graph the function for squares 1 through 10.**

4. Radium Decay

In 2000, 50 grams of radium were stored. The half-life of radium is 1,620 years. How many grams of radium remains after 4860 years? Remember, half-life is the amount of time it takes for half of the amount of a substance to decay.

a. Complete the chart:

End of Half LIfe Cycle	1620 yrs	3240 yrs	4860 yrs
Number of half life cycles	1	2	3
Bacteria present			
Pattern			

b. Write a function to illustrate the situation.

c. Plot the data and graph the function.

Answer to Question 1:

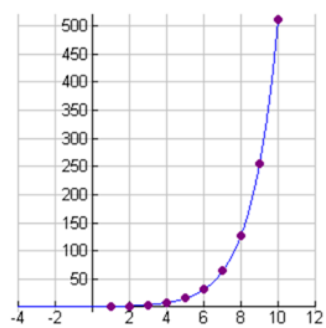
a. Complete the chart:

Number of the doubling	1	2	3	4	5	6	7	8	9	10	11	12	13	...	63
Wheat on each square	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	...	9,223,372,036,854,775,808
Pattern:	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}		2^{63}

FYI: In total, the king placed 18,446,744,073,709,551,615 grains of wheat on the board. This is more wheat than exists in the entire world. China is the largest producer of wheat, producing approximately 3.8 billion bushels per year. It would take China well over 6000 years to fill the 64 squares on the chess board.

b. Function: $y = 1(1+1.00)^x$ From Pattern: $y = 2^x$

c. Graph: *horizontal axis = square on board*
vertical axis = wheat



Answer to Question 2:

a. Complete the chart:

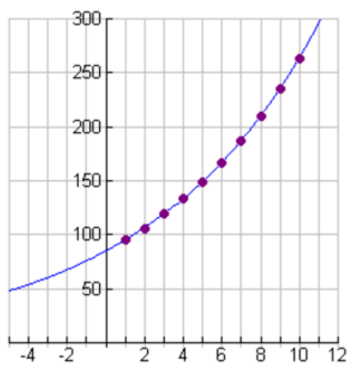
Years	1 1996	2 1997	3 1998	4 1999	5 2000	6 2001	7 2002	8 2003	9 2004	10 2005
Number of Rabbits (Round to the nearest rabbit.)	95	106	119	133	149	167	187	210	235	263

b. Function: $y = 85(1+.12)^x$

c. Plot the data and graph the function.

horizontal axis = year (1 = 1996)

vertical axis = number of rabbits



Answer to Question 3:

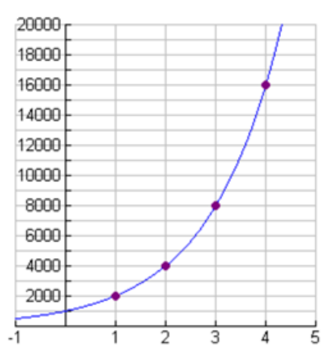
a. Complete the chart:

Time intervals 30 minutes	1 .5 hr	2 1 hr	3 1.5 hr	4 2 hr	5 2.5 hr	6 3 hr	7 3.5 hr	8 4 hr	9 4.5 hr	10 5 hr
Bacteria present	2000	4000	8000	16000	32000	64000	128000	256000	512000	1024000
Pattern:	$1000*2^1$	$1000*2^2$	$1000*2^3$	$1000*2^4$	$1000*2^5$	$1000*2^6$	$1000*2^7$	$1000*2^8$	$1000*2^9$	$1000*2^{10}$

b. Function: $y = 1000(1+1.00)^x$ From pattern: $y = 1000 \bullet 2^x$

c. Graph: *horizontal axis = 30 min. time intervals*

vertical axis = number of bacteria



d. Find the number of bacteria present after 45 minutes.
From looking at the data table: 45 minutes is half way between 30 minutes and one hour. If this process were "linear" we could make an estimate of the bacteria to be half way between 2000 and 4000 which would be 3000 bacteria. However, exponential growth is not linear. If you examine the graph, the number of bacteria at 45 minutes equals 2828.4271 bacteria.

Answer to Question 4:

a. Complete the chart:

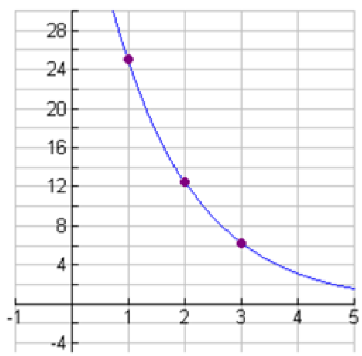
End of Half life cycle	1 1620 yrs	2 3240 yrs	3 4860 yrs
Grams of radium remaining	25	12.5	6.25
Pattern:	$\frac{50}{2^1}$	$\frac{50}{2^2}$	$\frac{50}{2^3}$

b. Function: $y = 50(1-.5)^x$ From the pattern: $y = \frac{50}{2^x}$

c. Plot the data and graph the equation for the first 3 time intervals.

horizontal axis = end of half life cycle

vertical axis = grams of radium remaining



Original was posted at http://regentsprep.org/Regents/math/ALGEBRA/AE7/ExpDecayP.htm and version is available at https://web.archive.org/web/20160425022318/http://regentsprep.org/Regents/math/ALGEBRA/AE7/ExpDecayP.htm