Weaving a Parabola Web with the Quadratic Transformer

In this activity, you explore how the graph of a quadratic function and its symbolic expression relate to each other. You start with a set of four graphs, which we'll call a Parabola Web.

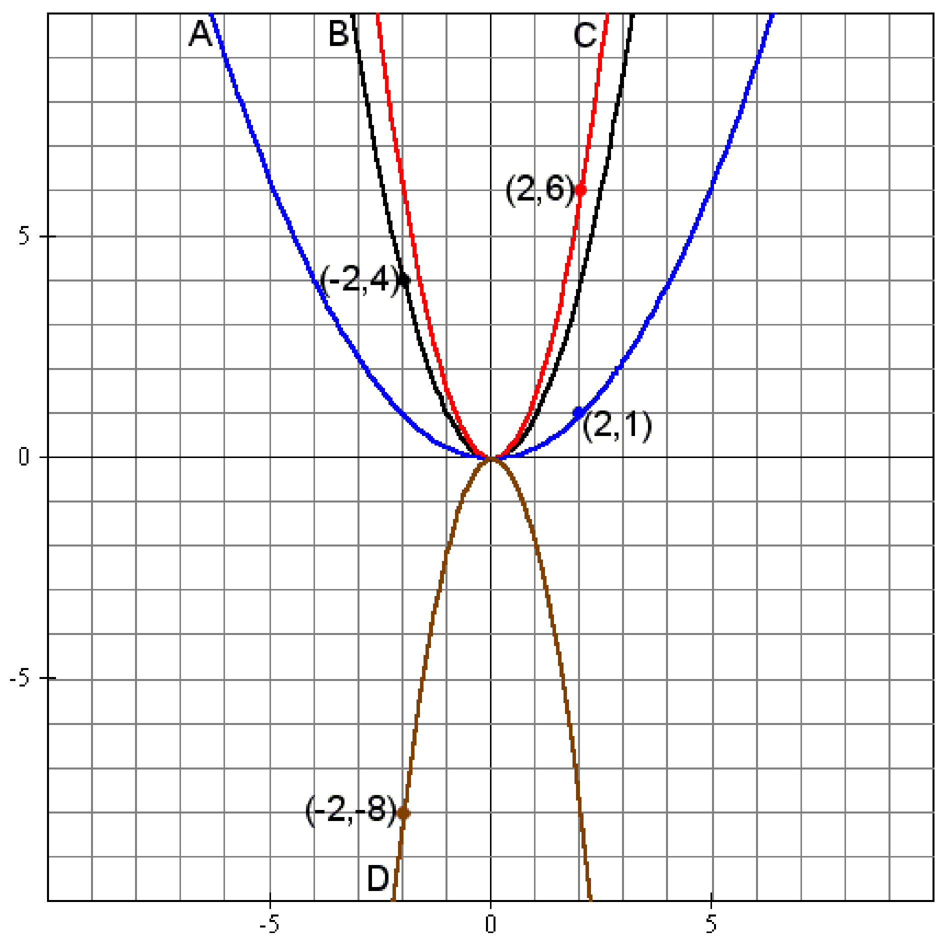
To graph and manipulate the parabolas, use the Quadratic Transformer available at http://seeingmath.concord.org/resources_files/QuadraticGeneral.html.

Before You Start

Get acquainted with the computer program, *The Quadratic Transformer*. When you're satisfied that you understand how it works, try the tasks below.

Challenge A: What About the Coefficient?

Look at the Parabola Web. All the parabolas pass through (0, 0). The function rules that describe each parabola have the form $y = ax^2$, where a is a constant.



Challenge A (continued)

1. Use the *Quadratic Transformer* to help you write the function rule for each of the parabolas in the web.

Look closely at the rules in Problem 1.

In problems 2–4, try to figure out how the coefficient of x^2 (the letter a) is related to the graph of the corresponding parabola.

2. Write a function rule to describe a parabola with the same vertex, whose other points lie somewhere between parabolas A and B.

3. Write a function rule to describe a parabola that looks like parabola B reflected about the x-axis.

4. Write a function rule to describe a parabola that looks like parabola D reflected about the x-axis.

Challenge A (continued)

5. Now that you've investigated with the *Quadratic Transformer*, how does the number you chose for the coefficient of x^2 (the letter a) change the shape of a parabola? Write your conclusions and explain your reasoning.

Challenge B: Moving the Parabolas

The original parabolas form a web with vertices at (0, 0).

1. Show how you would move the vertex of each parabola from (0, 0) to (0, 1) by changing the function rule. Moving a graph in this way is called a *vertical translation*.

2. Show how you would move the vertex of each parabola from (0, 0) to (2, 0). Moving a graph in this way is called a *horizontal translation*.

3. Show how you would move the vertex of each parabola from (0, 0) to (-1, 2). Demonstrate and explain your reasoning using just one of the parabolas. How does the function rule for the parabola change?

Challenge C: Comparing Forms

In the following problems, compare and contrast what the polynomial and root forms of a quadratic function can tell you about its graph.

What are the forms of a quadratic equation?

POLYNOMIAL FORM: $y = ax^2 + bx + c$

ROOT FORM: $y = a(x - m)(x - p)$

VERTEX FORM: $y = a(x - h)^2 + k$

Note: *For the purpose of this activity, assume that the coefficients in front of the first term of both the root form and the vertex form are 1, as shown above.*

1. a. Graph these two functions:

   $y = x^2 - 6x + 5$ ⇦ Polynomial form

   $y = (x - 2)(x + 3)$ ⇦ Root form

   b. Where does each parabola cross the x-axis, if at all? Where does each parabola cross the y-axis, if at all?

   c. Which of the two forms for a quadratic function tells you more about where a function crosses the x-axis? The y-axis? Explain.

Challenge C (continued)

In problems 2–3, refer back to the Parabola Web.

2. a. Translate parabola B so that it crosses the *x*-axis at two points: (-2, 0) and (2, 0). Record your rule.

   b. Now graph a new parabola that crosses the *x*-axis at the same two points: (-2, 0) and (2, 0). Record your rule.

   c. How are the rules that describe these two parabolas similar? How are they different? Which symbolic form was more useful for this problem?

3. a. Translate parabola D so it crosses the *y*-axis at (0, -8). Record your rule.

   b. Now translate parabola D again, so that it crosses the *y*-axis at a different point of your choice. Record your rule.

   c. Which form of the rule tells you more about where a graph crosses the *y*-axis?