Zeros of a Quadratic Function

MATH NSPIRED

Math Objectives

- Students will determine the linear factors of a quadratic function.
- Students will connect the algebraic representation to the geometric representation.
- Students will discover that the zeros of the linear factors are the zeros of the quadratic function.
- Students will discover that the zeros of the quadratic function are the zeros of its linear factors.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

zeros

About the Lesson

- This lesson involves merging graphical and algebraic representations of a quadratic function and its linear factors.
- As a result, students will:
 - Manipulate the parameters of the linear functions and observe the resulting changes in the quadratic function.
 - Find the zeros of the quadratic function by finding the zeros of its linear factors.
 - Solve quadratic equations by factoring and be able to explain why this process is valid.

TI-Nspire[™] Navigator[™] System

- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing
 ctrl G.

Lesson Materials:

Student Activity Zeros_of_a_Quadratic_Function _Student.pdf Zeros_of_a_Quadratic_Function Student.doc

TI-Nspire document Zeros_of_a_Quadratic_Function .tns

Visit <u>www.mathnspired.com</u> for lesson updates and tech tip videos.

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Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point on the slider, check to make sure that they have moved the arrow close to the point. The arrow should become a hand (ⓐ) getting ready to grab the point. Press **ctrl** ⓐ to grab the point, and the hand will close. Drag the point. After the point has been moved, press **esc** to release the point.

Move to page 1.2.

 Use the sliders to set y₁ = 2x + 2 and y₂ = 1x - 2. Observe that the graph of y₁ = 2x + 2 appears to cross the x-axis at x = -1. When x = -1, y₁ = 0 because 2(-1) +2 = 0.

x = -1 is called a *zero* of the function $y_1 = 2x + 2$.

a. Where does the graph of $y_2 = 1x - 2$ appear to cross the x-axis?

Answer: at *x* = 2

b. Verify that this value of x is a zero of y_2 .

<u>Answer:</u> 1(2) – 2 = 0

Teacher Tip: A zero of a function is an input value for which the function value is zero. Thus, if x = 2 is a zero of a function, then f(2) = 0, and the point (2, 0) is on the graph of the function.

2. a. When $y_1 = 2x + 2$ and $y_2 = 1x - 2$, what is the function y_3 ?

Answer: $y_3 = 2x^2 - 2x - 4$

b. How many times does the graph of $y_3 = 2x^2 - 2x - 4$ cross the x-axis?

Answer: The graph crosses the *x*-axis twice.



c. What are the zeros of y_3 ?

Answer: *x* = –1 and *x* = 2

d. Write a conjecture about the relationship between the zeros of the linear functions and the zeros of the quadratic function.

Answer: The zeros of the linear functions are the zeros of the quadratic function.

TI-Nspire Navigator Opportunity: Screen Capture	
See Note 1 at the end of this lesson.	

Teacher Tip: If students have trouble writing a conjecture to answer question 2d, ask what they notice about the graphs of the functions. Remind them that a zero of a function is a place where f(x) = 0, or a place where the graph crosses the *x*-axis.

3. a. Given the information below, use the sliders of the .tns document to fill in the rest of the table.

Answer:

y 1	y 2	Zeros of		, v	Zeros of
		y 1	y 2	y 3	y 3
2 <i>x</i> + 4	<i>x</i> – 1	-2	1	$2x^2 + 2x - 4$	–2 and 1
3 <i>x</i> + 3	x + 4	-1	-4	$3x^2 + 15x + 12$	–1 and –4
x + 5	x – 4	-5	4	$x^2 + x - 20$	–5 and 4
x – 5	x + 3	5	-3	$x^2 - 2x - 15$	5 and –3

b. What is the relationship between the zeros of the quadratic function and the zeros of the linear functions? Compare this to the conjecture you made in question 2d.

Sample Answers: In each row, the quadratic function has the same zeros as the two linear functions. This is the same as my conjecture in question 2d.

4. Factor each of the quadratic functions below.

a. $2x^2 + 2x - 4$

<u>Answer:</u> (2x + 4)(x - 1)

b. $3x^2 + 15x + 12$

<u>Answer:</u> (3x + 3)(x + 4) = 3(x + 1)(x + 4)

c. $x^2 + x - 20$

<u>Answer:</u> (x + 5)(x - 4)

d. $x^2 - 2x - 15$

<u>Answer:</u> (x - 5)(x + 3)

5. How do the factors in question 4 relate to the information in the table in question 3?

Answer: The factors of the quadratics can be the same as the linear function in the same row.

 Write a pair of linear functions whose product yields a quadratic function with zeros of 3 and -2. What is the corresponding quadratic function? Describe the process you used to determine your answers.

Answer: The pair of linear functions is $y_1 = x - 3$ and $y_2 = x + 2$. The quadratic function is $y = x^2 - x - 6$.

The linear functions are found by $y - z_1$ and $y - z_2$, where z_1 and z_2 are the zeros supplied, and the quadratic function is the product of the two linear functions: $(y - z_1)(y - z_2)$.

TI-Nspire Navigator Opportunity: *Live Presenter* See Note 2 at the end of this lesson.

7. Given the quadratic function $y = x^2 - 11x + 30$, determine its zeros. Describe the process you used to obtain your solutions.

<u>Answer:</u> y = (x - 6)(x - 5)x = 6 and x = 5

First, factor the quadratic function into its two linear factors. Then find the zeros of each of the linear factors. These are also the zeros of the quadratic function.



Teacher Tip: This quadratic function is intentionally chosen so that students will not be able to model it in the .tns document. They must factor the function to find its zeros. It might be necessary to ask directed questions to help students understand that they need to factor the quadratic into its linear factors and then find those zeros.

8. Samuel says, "I can solve $x^2 - 11x + 30 = 0$ by factoring it, setting each factor equal to zero, and solving for *x*." Is this a valid method? Explain.

<u>Answer:</u> Yes. Setting each factor to zero finds the zeros of the two linear factors which we know to be the zeros of the quadratic.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use a graph to find possible linear factors of a quadratic function.
- The connections between the algebraic and graphical representations of a quadratic function and its factors.
- The zeros of the linear factors of a quadratic function and the zeros of the quadratic function are the same.
- The zeros of a quadratic function are the same as the zeros of its linear factors.

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Note 1

Questions 1 and 2, *Screen Capture*: Use the Screen Capture feature to observe all of the screens at once to ensure that students are having success and seeing the correct graphs for questions 1 and 2. Assist students who are having difficulty.

Note 2

Question 6, *Live Presenter*: Pick a different student to present his or her findings for the answers to question 3a. Then have a different student illustrate his or her answer to question 6.

Optional: After discussing question 6, have students do the following exercise: *Write a pair of linear functions whose product yields a quadratic function with zeros 4 and –1. What is the corresponding quadratic function? Describe the process you used to determine your answers.* Use Screen Capture to see how students use the sliders on page 1.2 to answer this question. If an explanation is needed, choose a student who was successful and make that student the Live Presenter and explain the solution.