ELD STANDARD 3: The Language of Mathematics	MAISA Algebra I, Unit 5: Solving Quadratic Equations

CONNECTIONS: Michigan Academic Standards for Mathematics - Algebra I

EXAMPLE CONTEXT FOR LANGUAGE USAGE: The listening demand is high during this lesson launch and summary. The strand below supports the listening students will do to be engaged in classroom conversation during the lesson launch while using visual supports to understand the algorithm for completing the square. One of the supports listed is a glossed version of the worksheet. While this is technically a reading support, the students will rely on the worksheet in order to synthesize the classroom discussion. The purpose for listening in this activity is for students to understand the visual representation of the algorithm for completing the square so they can apply the algorithm visually in the next step of the lesson. Note that the example in the strand includes sample language from a classroom conversation where the teacher and several students are modeled. During the conversation, which occurs immediately after Part A, the teacher records a list of criteria for students to apply in Part B of the worksheet. Refer to "Changing Forms Student Activity Sheet" in the supporting documents.

COGNITIVE FUNCTION: Students at all levels of English language proficiency will **SYNTHESIZE** information heard orally in order to **APPLY** strategies to problem solving.

SAMPLE CLASSROOM CONVERSATION: (see ListeningAreaModelsCriteria: The goal is for students to connect the idea of creating perfect squares to convert from standard form to vertex form. Students don't yet know the algorithm which means they need to notice any quadratic can be written as a perfect square added to some quantity whether 0 or nonzero)

T: [As students participate in class dialogue the teacher will record criteria that students will use throughout the lesson.] You just created area models [point to area models on anchor chart] for quadratics given in factored form [point to anchor chart]. How were the equations in questions 3, 4, and 6 different from the others? [Teacher uses document camera to highlight these numbers on the worksheet]
S1: Questions 3, 4, and 6 each have two equal factors. Questions 1, 2, and 5 each have two unequal factors.
T: You said they have equal factors [points to factors on the anchor chart, then record on criteria list]. How did having equal factors affect the area models?
S2: The dimensions of the rectangle are the equal.
S3: What do you mean by dimensions?
S2: The dimensions are the length and width of the rectangle. [Teacher records on criteria list.]
S4: There were the same number of *x* tiles (longs) below and to the right of the *x*^2 tile (square).
T: Can someone restate what S4 said while pointing to an area model? [Another student restates with pointing to model.] Are there other reasons why 3, 4, and 6 are different?
S5: They formed a square, which makes sense because they have the same dimensions. [Teacher records on criteria list.]
T: Can someone show me one of the representations where S5 sees a square?
S6: [Pointing to the area model] Since they have the same length and width, they form a square.
S7: I also see a square formed by the little unit tiles. [Teacher records on criteria list.]
T: Can you show us? [Student points to model.] A quadratic with equal factors is in factored form and vertex form at the same time: *y* = (*x* - *h*)^2 + *k* = *(x - h)* ^2 + 0 = (x - h)^2. In Part B use the criteria we came up with as a class. [point to the list of criteria] Draw pictures to change each quadratic equation from standard form [point to anchor chart] to vertex form [point to anchor chart].

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Synthesize information from a classroom conversation in order to apply strategies for completing the square visually when the teacher has paused and pointed to classroom anchor chart and asked students to restate or connect to a concrete model to explain their thinking and using the worksheet and a student- or teacher-created vocabulary reference sheet (includes terms shown in anchor chart example). (While the level 1 students may not benefit from additional supports during the listening portion of the lesson, they would benefit from working with a partner during the activity following the classroom discussion.)	Synthesize information from a classroom conversation in order to apply strategies for completing the square visually when the teacher has paused and pointed to classroom anchor chart and asked students to restate or connect to a concrete model to explain their thinking and using the worksheet and a student- or teacher-created vocabulary reference sheet (includes terms shown in anchor chart example).	Synthesize information from a classroom conversation in order to apply strategies for completing the square visually when the teacher has paused and pointed to classroom anchor chart and asked students to restate or connect to a concrete model to explain their thinking and using the worksheet (includes terms shown in anchor chart example).	Synthesize information from a classroom conversation in order to apply strategies for completing the square visually when the teacher has paused and pointed to classroom anchor chart and asked students to restate or refer to a concrete model to explain their thinking.	Synthesize information from a classroom conversation in order to apply strategies for completing the square visually when the teacher has paused and pointed to classroom anchor chart and asked students to restate or refer to a concrete model to explain their thinking.	

ELD STANDARD 3: The Language of Mathematics	MAISA Algebra I, Unit 5: Solving Quadratic Equations

EXAMPLE CONTEXT FOR LANGUAGE USAGE: This lesson would take place toward the beginning of a quadratic functions unit. In this speaking task, students will record themselves as they interpret he structure and parameters of a quadratic function (y=ax^2+bx+c) and its graph in order to assist in finding solutions. Students will describe and eventually predict the shape of a graph based on coefficients in a given formula. Students will have the "Multipurpose Support: Introduction to Quadratic Functions" document (see supports) to use as well, while creating and describing graphs.

COGNITIVE FUNCTION: Students at all levels of English language proficiency will successfully **DESCRIBE** and **PREDICT** what a function's graph will look like based on analyzing the formula.

Resources:

https://www.littlebirdtales.com/ The Little Bird Tales app (or any similar app) can be used to allow students to record their own voices as they describe and predict in this task.

https://www.desmos.com/calculator The Desmos calculator (or any similar app) can be used to allow students to graph functions quickly to test their conjectures.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking	Justify in simple sentences a prediction of the function's graph based on the parameters of a quadratic function in standard form using glossed sentence frames with some answer choices, working with a level 2 partner, and rehearsing for the teacher or an English proficient peer prior to recording. E.g., [Given the prompt: Predict what the graph of y = 2x^2 - 4x + 5 will look like.] See sample student response in Speaking_ LittleBirdTalesSample The parabola opens _______ (up/down) because ___(a/b/c) is______(positive/negative). Compare to *y* = *x*^2. The parabola y = 2x^2 - 4x + 5 is steeper ___(a/b/c) is ______(bigger/equal to/smaller) than 1. I find the vertex. h = ___ and k = ___. The parabola will shift [gloss: move] _____(#) units ________ (up/down) because_______. The parabola will shift [gloss: move] _____(#) units to the _______________(right/left) because_______.	Justify in simple sentences a prediction of the function's graph based on the parameters of a quadratic function in standard form using glossed sentence frames with some answer choices, working with a level 1 partner, and rehearsing for the teacher or an English proficient peer prior to recording. E.g., [Given the prompt: Predict what the graph of *y = 2x^2 - 4x + 5* will look like.] See sample student response in Speaking_ LittleBirdTalesSample The parabola opens _______ (up/down) because ___(a/b/c) is______(positive/negative). Compare to y = x^2. The parabola y = 2x^2 - 4x + 5 is steeper ___(a/b/c) is ______(bigger/equal to/smaller) than 1. I find the vertex. h = ___ and k = ___. The parabola will shift [gloss: move] _____(#) units ________ (up/down) because_______. The parabola will shift [gloss: move] _____(#) units to the _______________(right/left) because_______.	Justify in simple sentences a prediction of the function's graph based on the parameters of a quadratic function in standard form using a suggested word list (e.g., parabola, quadratic function, open up/down, shifted, positive/negative, stretch/shrink, y-intercept, x-intercept) and checking work with a partner. E.g., [Given the prompt: Predict what the graph of y = 2x^2 - 4x + 5 will look like.] See sample student response in Speaking_ LittleBirdTalesSample	Justify in compound and/or complex sentences a prediction of the function's graph based on the parameters of a quadratic function in standard form and checking work with a partner. E.g., [Given the prompt: Predict what the graph of y = 2x^2 - 4x + 5 will look like.] See sample student response in Speaking_ LittleBirdTalesSample	Justify in compound and/or complex sentences a prediction of the function's graph based on the parameters of a quadratic function in standard form and checking work with a partner. E.g., [Given the prompt: Predict what the graph of *y = 2x^2 - 4x + 5* will look like.] See sample student response in Speaking_ LittleBirdTalesSample	

ELD STANDARD 3: The Language of Mathematics				MAISA Algebra I, Unit 5: Solving Quadratic Equations		

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students read and interpret a mathematical description that can be modeled by a quadratic function y = ax^2 + bx + c and use the parameters of the function (a, b, and c) to assist in determining an appropriate method to solve the equation and analyzing the relationship between the number of solutions and the maximum/minimum value of the function. For example, if there are two solutions, then the maximum or minimum value of the quadratic function will lie between those two solutions; the maximum or minimum value will be the y-value of the horizontal midpoint between the two solutions. If there is only one solution, that solution represents the location of the maximum/minimum value. A general illustration of the scenario may be appropriate for all language levels when the context is unfamiliar.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **INTERPRET** a written mathematical description in order to **DETERMINE** an appropriate method of solving the equation and **ANALYZE** the significance of the number of solutions.

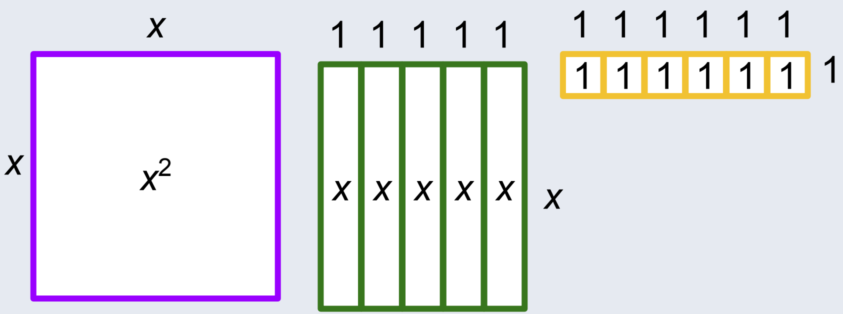
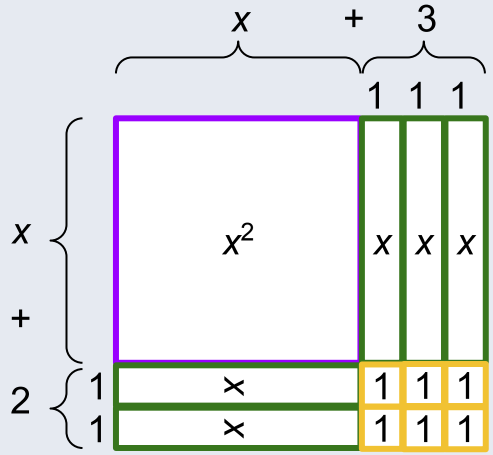
	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Interpret a linguistically complex mathematical description that can be modeled by a quadratic function in order to solve an equation and answer questions regarding the relationship between the number of solutions and the maximum/minimum value of the function, using a glossed and illustrated version of the text and working with a small group of students with similar or higher language proficiency. E.g., An expert [gloss: very good] golfer [refer to a picture of a golfer swinging a golf club] hits a golf ball up from the ground [refer to a picture of a golf club hitting a ball] with an initial [gloss: beginning] upward velocity [gloss: speed] of 100 feet per second. Use the vertical [gloss: up and down] motion [gloss: movement] equation h = -16t^2 + 100t to find when the ball will be at a height of 136 feet. Does the ball travel [gloss: go] higher than 136 feet? How do you know?	Interpret a linguistically complex mathematical description that can be modeled by a quadratic function in order to solve an equation and answer questions regarding the relationship between the number of solutions and the maximum/minimum value of the function, using a glossed and illustrated version of the text and working in a small group of students with similar language proficiency. E.g., An expert [gloss: very good] golfer [refer to a picture of a golfer swinging a golf club] hits a golf ball up from the ground [refer to a picture of a golf club hitting a ball] with an initial [gloss: beginning] upward velocity [gloss: speed] of 100 feet per second. Use the vertical [gloss: up and down] motion motion [gloss: movement] equation h = -16t^2 + 100t to find when the ball will be at a height of 136 feet. Does the ball travel [gloss: go] higher than 136 feet? How do you know?	Interpret a linguistically complex mathematical description that can be modeled by a quadratic function in order to solve an equation and answer questions regarding the relationship between the number of solutions and the maximum/minimum value of the function using a glossed version of the text and working with a partner. E.g., An expert [gloss: very good] golfer hits a golf ball up from the ground with an initial [gloss: beginning] upward velocity [gloss: speed] of 100 feet per second. Use the vertical [gloss: up and down] motion [gloss: movement] equation h = -16t^2 + 100t to find when the ball will be at a height of 136 feet. Does the ball travel [gloss: go] higher than 136 feet? How do you know?	Interpret a linguistically complex mathematical description that can be modeled by a quadratic function in order to solve an equation and answer questions regarding the relationship between the number of solutions and the maximum/minimum value of the function, working with a partner. E.g., An expert golfer hits a golf ball up from the ground with an initial upward velocity of 100 feet per second. Use the vertical motion equation h = -16t^2 + 100t to find when the ball will be at a height of 136 feet. Does the ball travel higher than 136 feet? How do you know?	Interpret a linguistically complex mathematical description that can be modeled by a quadratic function in order to solve an equation and answer questions regarding the relationship between the number of solutions and the maximum/minimum value of the function, checking work with a partner. E.g., An expert golfer hits a golf ball up from the ground with an initial upward velocity of 100 feet per second. Use the vertical motion equation h = -16t^2 + 100t to find when the ball will be at a height of 136 feet. Does the ball travel higher than 136 feet? How do you know?	

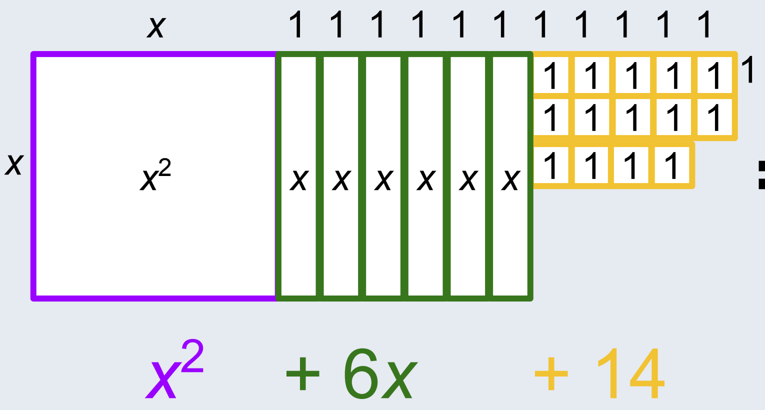
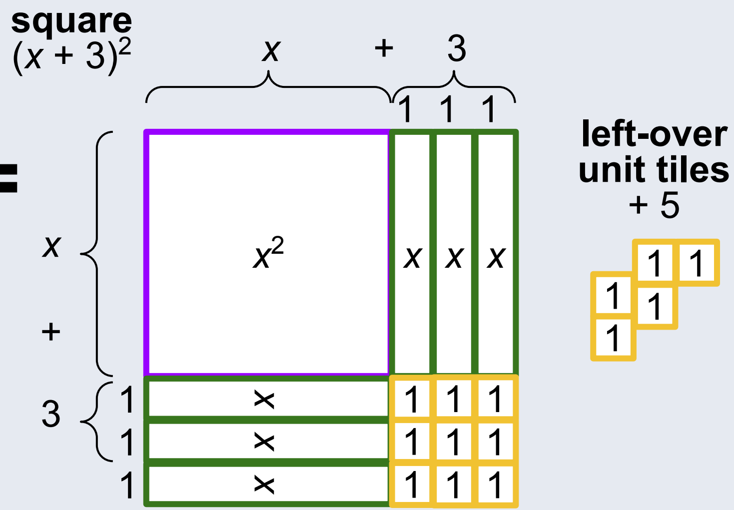
ELD STANDARD 3: The Language of Mathematics				MAISA Algebra I, Unit 5: Solving Quadratic Equations		

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students explain the relationship between the number of real roots and the graph of a quadratic relationship. The strand below is written as though students are making a final conclusion about this relationship after examining a set of quadratic functions. Similar conversations and supports could also be used if students were analyzing individual functions. E.g, "The graph of this function has two x-intercepts, so the function has two real roots." (Please note, students should be able to change the directionality of the connections from intercepts to roots. I.e., "Since the quadratic has 1 real root, its graph will have one x-intercept.") The strand below also suggests using conditional statements with the if/then structure. For students needing support with conditional statements, an "if-then" graphic organizer is provided. Students would benefit from working through examples with the graphic organizer before being required to use the tool in producing language independently.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **DESCRIBE** how the number of real roots for a quadratic relationship affect the graph.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Writing or Speaking	Identify and describe in simple sentences and/or phrases the relationship between the number of real roots and the graph of a quadratic relationship using an illustrated word bank and sentence frames, and an "if-then" organizer while working with a partner. If it has _____(#) _____________, then it has _____(#) _____________. If it has _____(#) _____________, then it has _____(#) _____________. If it has _____(#) _____________, then it has _____(#) _____________.	Identify and describe in simple sentences the relationship between the number of real roots and the graph of a quadratic relationship using an illustrated word bank and sentence frames while working with a partner. If the _______(function/graph) has_____(#) _____________, then the__________ (function/graph) has _____(#) _____________. If the _______(function/graph) has_____(#) _____________, then the__________ (function/graph) has _____(#) _____________. If the _______(function/graph) has_____(#) _____________, then the__________ (function/graph) has _____(#) _____________.	Identify and describe in complete sentences the relationship between the number of real roots and the graph of a quadratic relationship using an illustrated word bank and suggested word list (e.g., if, then, function, real root(s)/zero(s), intercept(s), graph) and working with a partner. E.g., "If there are two real roots, then the graph has two x- intercepts. If there is one real root, then there is one x-intercept. If there are no real roots, then there are no x-intercepts."	Identify and describe in compound and/or complex sentences, the relationship between the number of real roots and the graph of a quadratic relationship using a suggested word list (e.g., if, then, function, real root(s)/zero(s), intercept(s), graph) working with a partner. E.g., "If the function has two real roots, then the graph has two x- intercepts. If the function has one real root, then the graph has one x- intercept. If the function has no real roots, then the graph has no x- intercepts."	Identify and describe in compound and/or complex sentences with transition words the relationship between the number of real roots and the graph of a quadratic relationship using a required word list (e.g., if, then, real root(s)/zero(s), intercept(s), graph) working with a partner. E.g., "If a function has two real roots/zeros, then its graph will have two x-intercepts. However, if a function has one real root, then the graph will have one x-intercept. Finally, if a function does not have any real roots, then the graph will not intersect the x-axis."	

	Standard Form		Factored Form
Algebraic Model	$\underline{x^2} + \underline{5x} + \underline{6}$ terms	=	$(\underline{x + 3})(\underline{x + 2})$ factors
Area Models	 x^2 + 5x + 6	=	

	Standard Form		Vertex Form
Algebraic Model	$\underline{x}^2 + \underline{6x} + \underline{14}$ terms	=	$(x + \underline{3})^2 + \underline{5}$ vertex: (-3, 5)
Area Models		=	

Changing Forms

Name________________

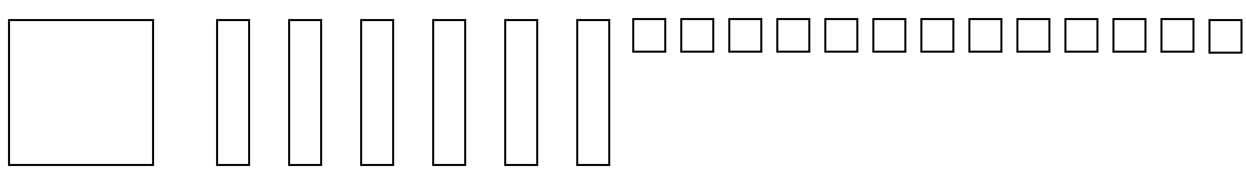
From Standard and Factored to Vertex Form

A. Draw an area model for each of the following products.

1. (x + 2)(x + 3)
2. (x+1)(x+4)
3. (x+2)(x+2)
4. (x + 1)(x + 1)
5. (x + 4)(x + 3)
6. $(x + 3)^2$
7. How were the equations in questions 3, 4, and 6 different from the others? What was special about their area models?

B. What happens when a quadratic is given in standard form? Is there a way to rewrite it in vertex form?

Write the quadratic represented below in standard form: ______________________



Rearrange the tiles to form a perfect square. (You may have some extra tiles.) Explain why vertex form of this quadratic is $(x+3)^2 + 4$.

Use algebra tiles to help complete the table below.

Standard Form	Sketch a picture of the vertex form	Vertex form
$x^2 + 8x + 19$		
$x^2 + 4x + 7$		
$x^2 + 2x + 7$		
$x^2 + 5x + 10$ *Hint*: $5x = 5/2\ x + 5/2\ x$		

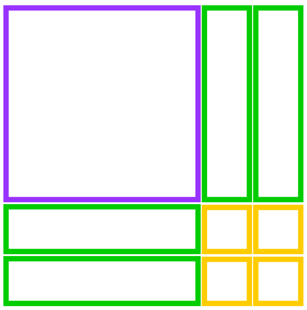
C. Describe in words the steps you used to convert each of the equations in standard form above to vertex form.

D. Use algebraic notation to summarize the process you use to complete the square for any quadratic of the form $x^2 + bx + c = 0$

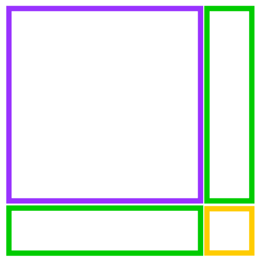
Criteria from Class Discussion

equal factors means …	
	…equal dimensions
	…length = width
	…number of rows = number of columns
area model is a **square** …	
	… because equal dimensions
	… because equal factors
unit tiles also form a square	

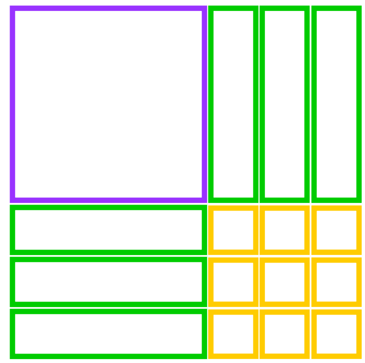
3. $(x+2)(x+2)$



4. $(x + 1)(x + 1)$



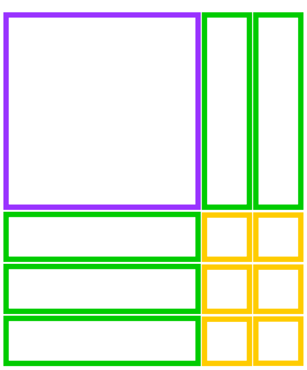
6. $(x + 3)^2$



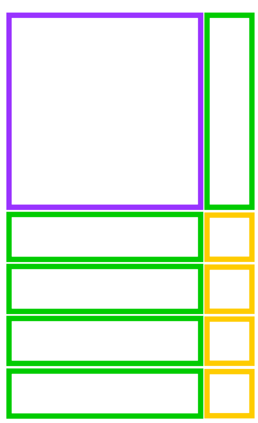
Vertex Form		
$y = (x + 2)^2 + 0$	$y = (x + 1)^2 + 0$	$y = (x + 3)^2 + 0$

unequal factors means area model is **not a square**	
	…unequal dimensions
	…length is different than width

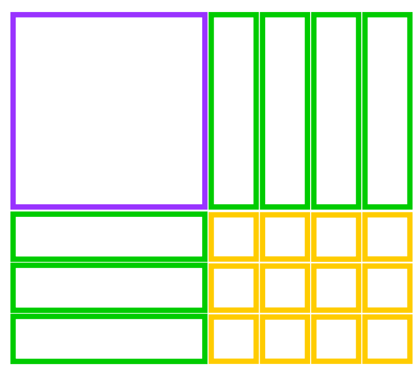
1. $(x + 2)(x + 3)$

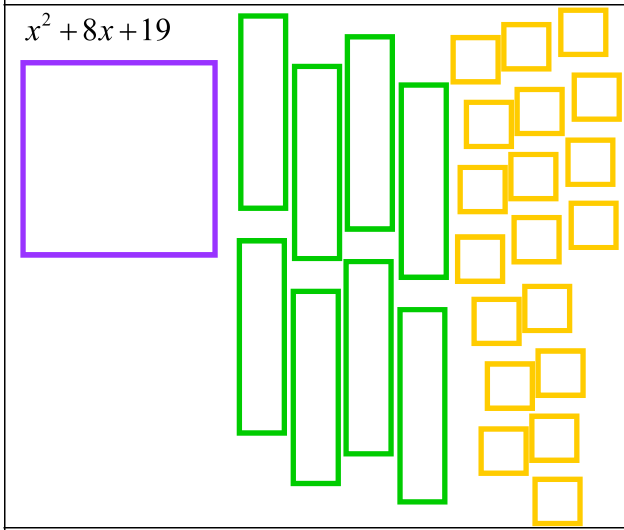
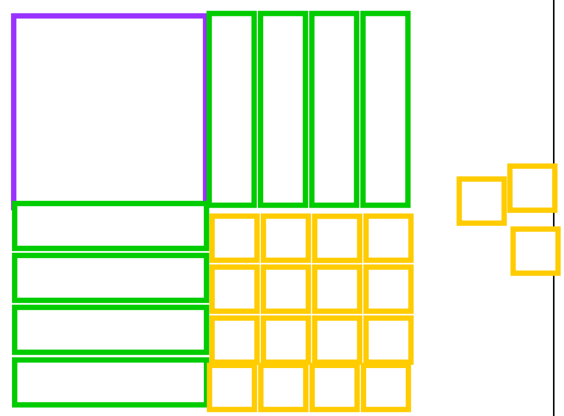
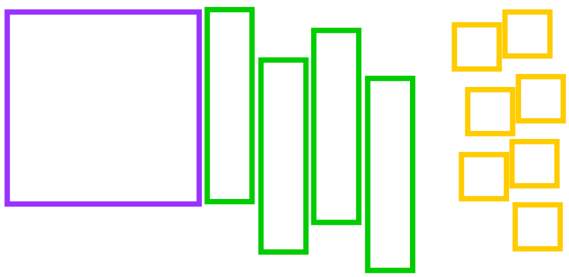
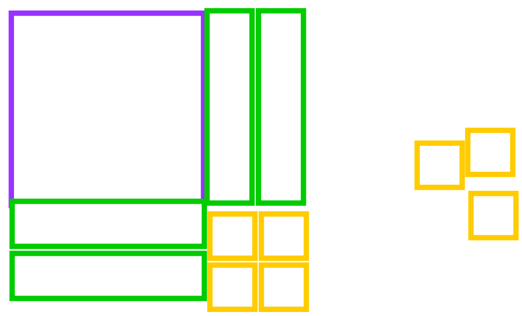
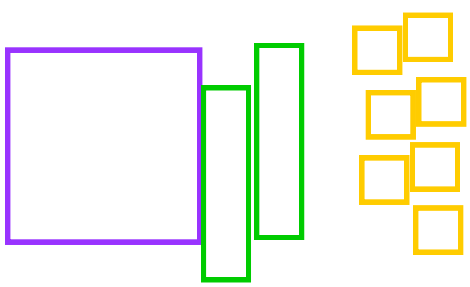
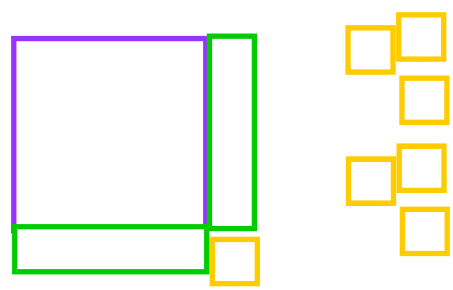
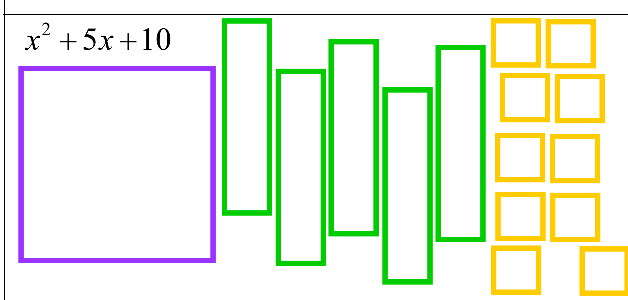
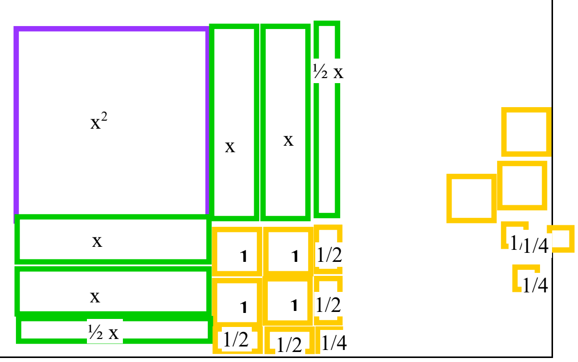


2. $(x+1)(x+4)$



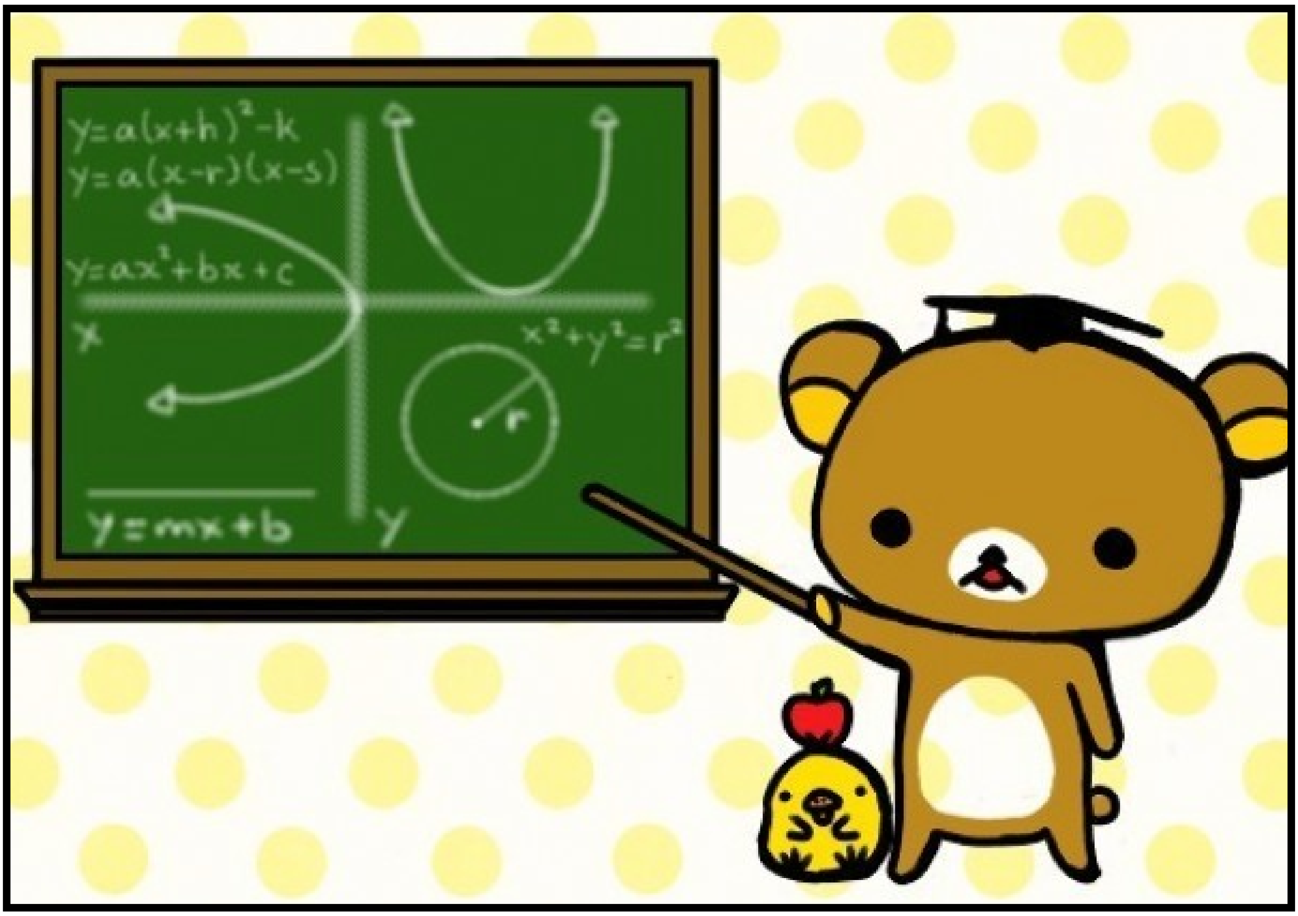
5. $(x + 4)(x + 3)$



Standard Form	Vertex form
$x^2+8x+19$ 	$(x+4)^2+3$ 
x^2+4x+7 	$(x+2)^2+3$ 
x^2+2x+7 	$(x+1)^2+6$ 
$x^2+5x+10$  ***Note to teachers: students might not realize they can break longs or unit tiles into fractional parts in their drawings, or how to do it effectively. After they experiment a while, make sure to address the idea in the summary.	$\left(x+\frac{5}{2}\right)^2+3\frac{3}{4}=(x+2.5)^2+3.75$ 

Ayanna's Tale

By Ayanna



Ayanna's Tale

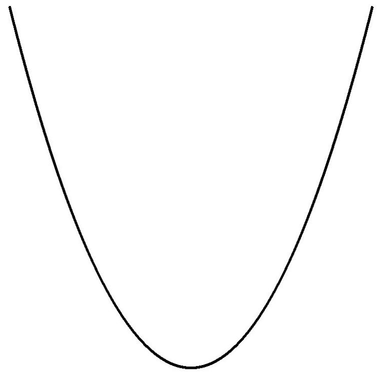
By Ayanna

Predict what the graph of

$$y = 2x^2 - 4x + 5$$

will look like

I know the parabola opens up
because *a* is positive 2.
If *a* was negative, it would open down.

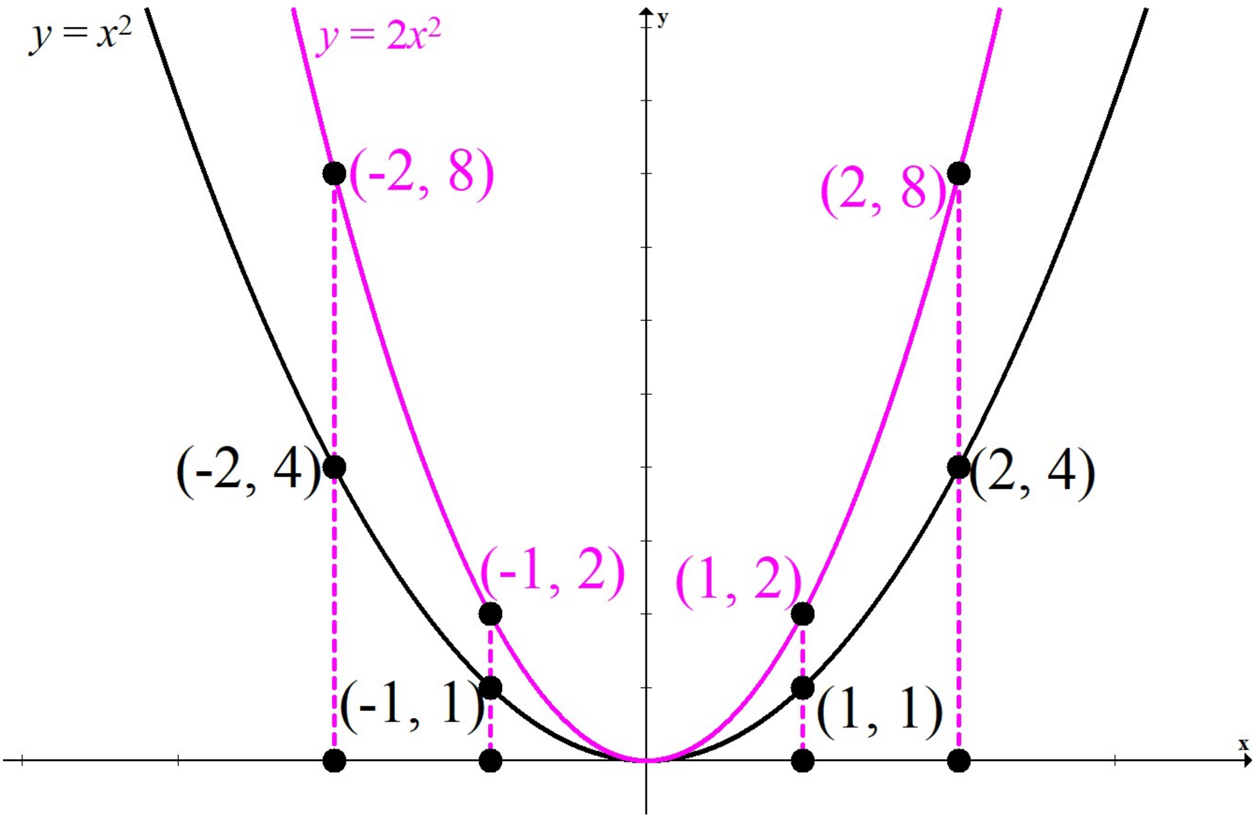


Ayanna's Tale

By Ayanna

I compare $y = x^2$ to $y = 2x^2$.

I know $y = 2x^2$ is steeper because $|a| > 1$.



Ayanna's Tale

By Ayanna

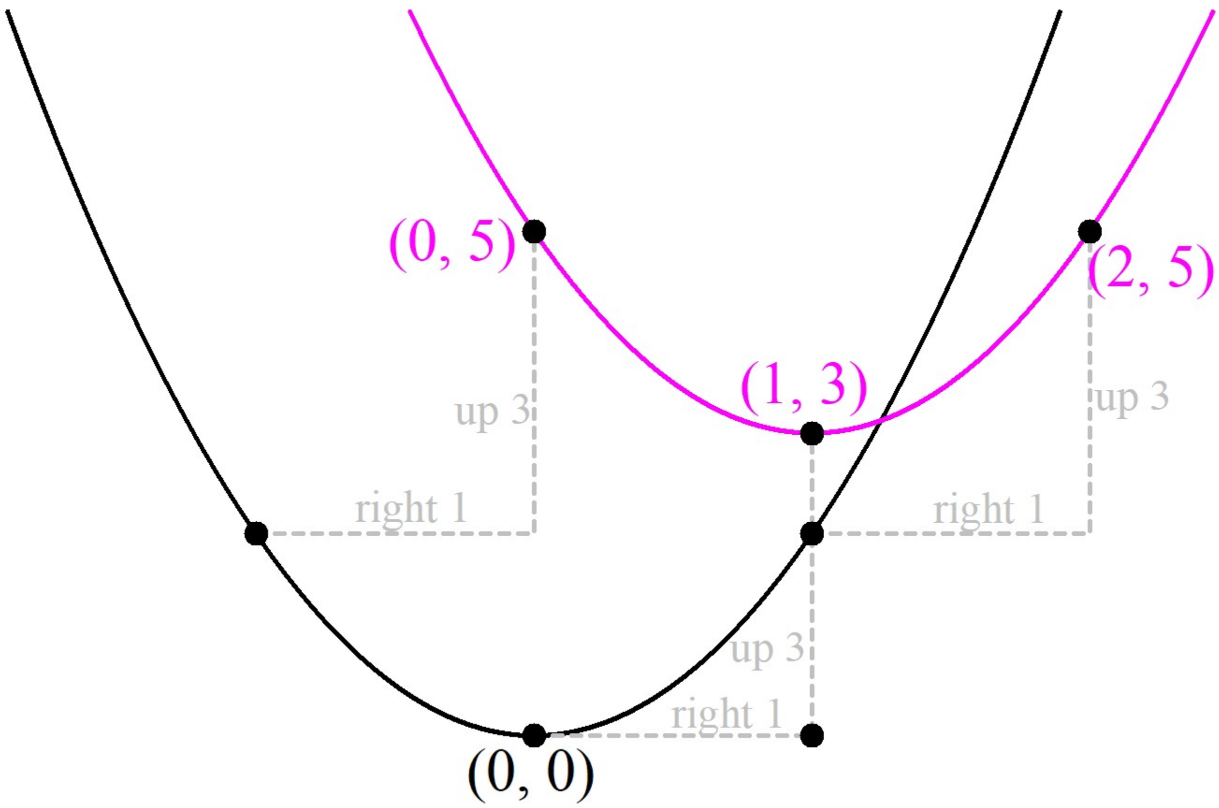
$y = 2x^2 - 4x + 5$

The vertex shows horizontal and vertical shift.

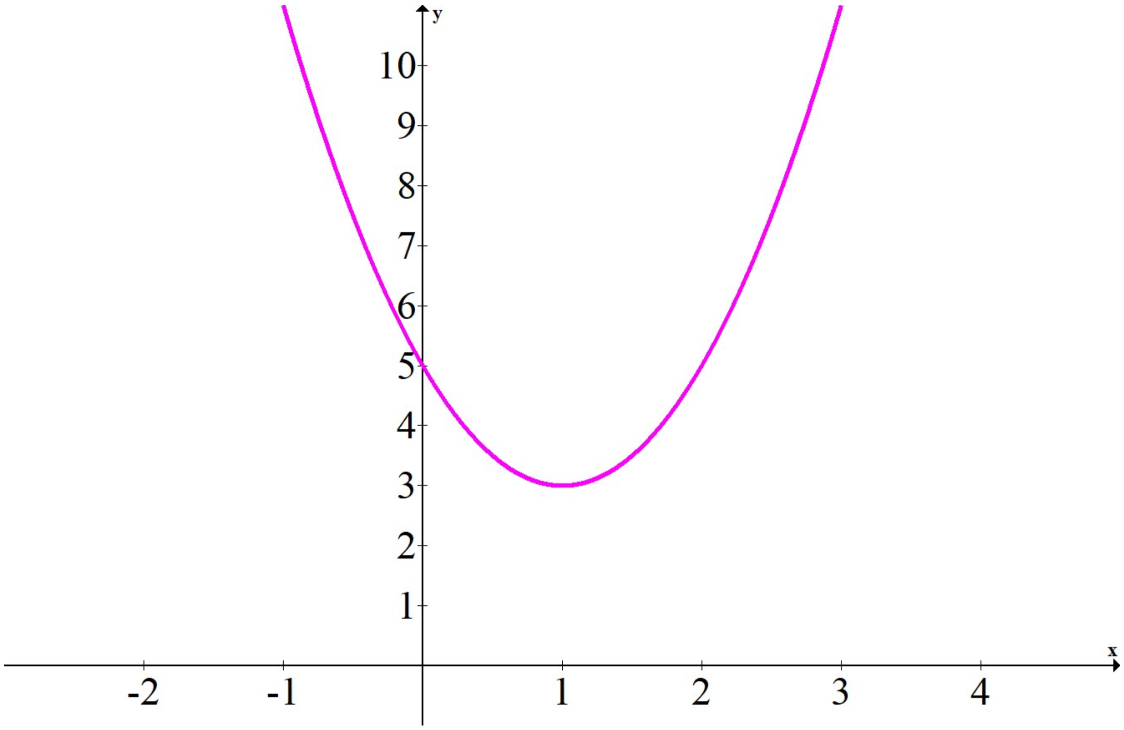
$$h = -\frac{b}{2a} = -\frac{-4}{2 \cdot 2} = 1$$

$$k = 2x^2 - 4x + 5 = 2(1)^2 - 4(1) + 5 = 2 - 4 + 5 = 3$$

$y = 2x^2$ shifts ***right by 1*** *$y = 2x^2$ shifts* ***up by 3***



The graph of $y = 2x^2 - 4x + 5$ is:



Algebra 1 Unit 5 Reading: Text with Support

Original text:

An expert golfer hits a golf ball up from the ground with an initial upward velocity of 100 feet per second. Use the vertical motion equation *h* = $-16t^2 + 100t$ to find when the ball will be at a height of 136 feet. Does the ball travel higher than 136 feet? How do you know?

Glossed and illustrated version of text:

Expert Golfer

https://en.wikipedia.org/wiki/Golf

[very good]

An <u>expert</u> golfer

hits a golf ball up from the ground

Golf Ball

https://en.wikipedia.org/wiki/Golf

[beginning] [speed]

with an <u>initial</u> upward <u>velocity</u> of 100 feet per second.

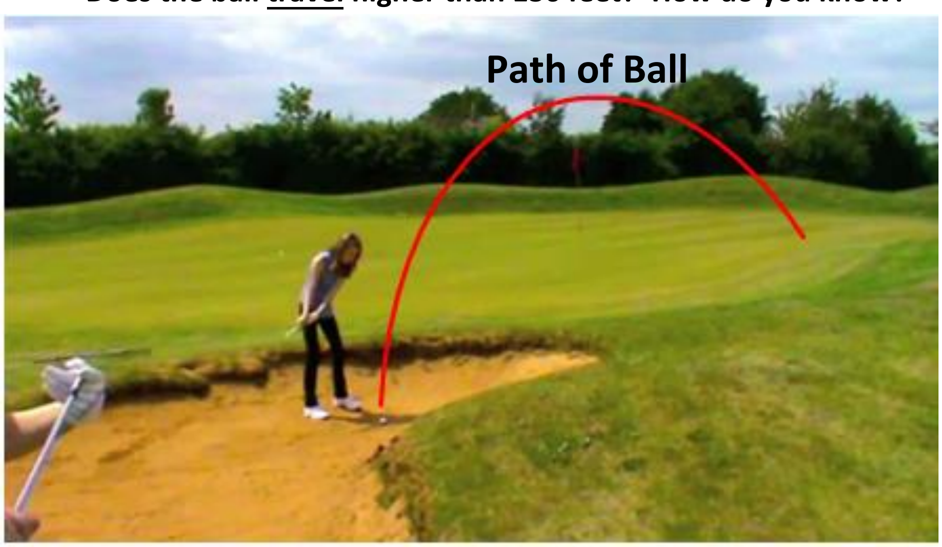
[up / down] [movement]

Use the <u>vertical</u> <u>motion</u> equation $h = -16t^2 + 100t$

to find when the ball will be at a height of 136 feet.

[go]

Does the ball <u>travel</u> higher than 136 feet? How do you know?



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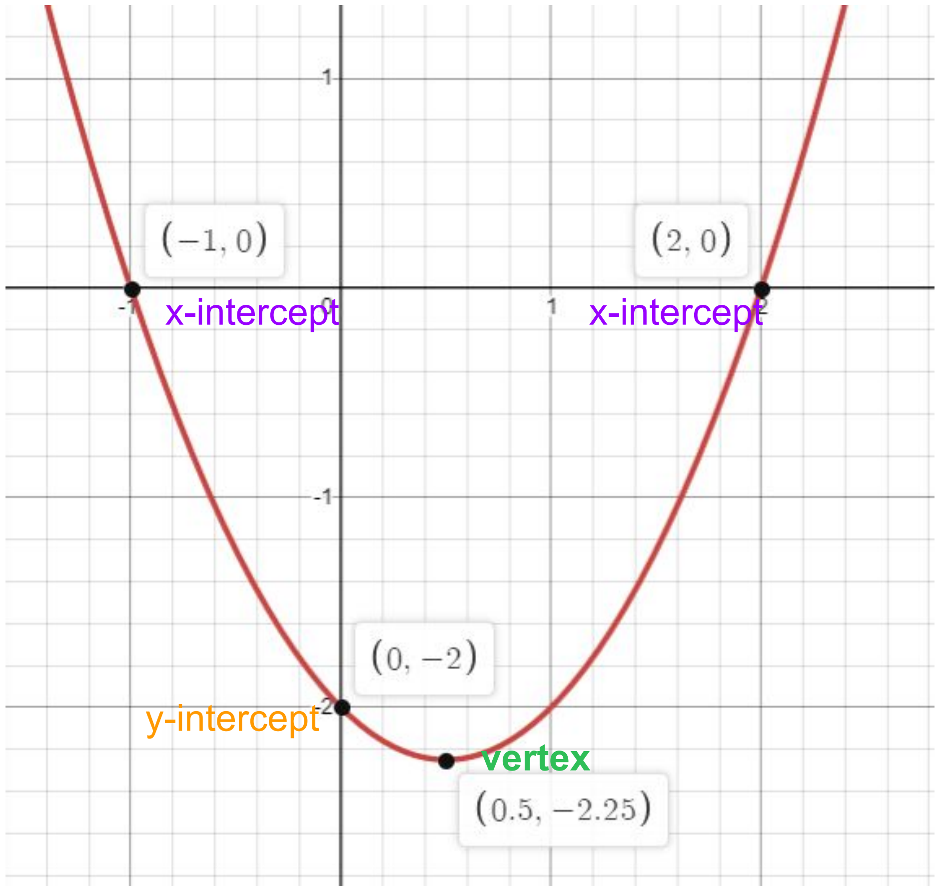


Graphical Features		**Algebraic Features**	**Related Equation** (equivalent equations for the same quadratic function)
x-intercepts	(-1,0) (2,0)	**Roots or Zeros** x-values that make the function value = 0 Substitute a root into any form of the equation for x, and the function (y-value) will be zero Example: *f(-1)= (-1+1)(-1+2)* *y= f(x) = (0) (1)* *y= f(x) = 0* **Factors** The factors of this quadratic are (x+1) and (x-2) When you set a factor = 0 and solve, you find the roots, x-intercepts, or zeros Example: *x+1= 0* *-1 -1* *x = -1*	**Factored Form** $y = (x+1)(x-2)$
y-intercept	(0,-2)	Substitute 0 into any form of the equation for x, and evaluate the function (find the y-value). $y = 0^2 - 0x - 2 = 0 - 0 - 2 = -2$	**Standard Form** $y = x^2 - 1x - 2$
vertex	(0.5, -2.25)		**Vertex Form** $y = 1(x-0.5)^2 - 2.25$

IF *(function)*		THEN (graph)
______ real roots (zeros) #	→	________ x-intercepts #

OR

IF (graph)		THEN *(function)*
______ x-intercepts #	→	_______ real roots (zeros) #

Option 1

If it has __ ______, then it has __ ______.
(intercepts/*roots/zeros*) # (intercepts/*roots/zeros*)

Option 2

If the _____ has __ ______, then there are __ ______.
(graph/*function*) # (intercepts/*real roots/zeros*) # (intercepts/real roots/zeros)

Unit 5 Introduction to the Quadratic Function Reference Sheet

Quadratic functions are second degree polynomials,
which means polynomials where the highest power on x is 2.

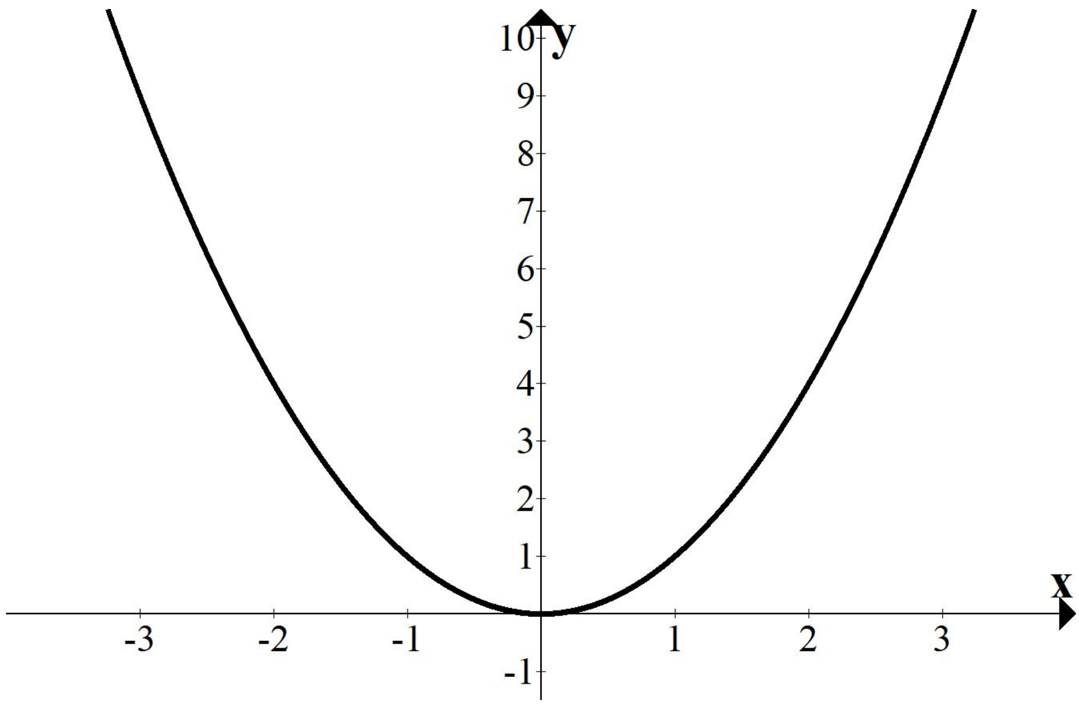
The graphical representation of a quadratic function is a curve called a parabola.

A symbolic representation can always be written $y = ax^2 + bx + c$,
where a is not 0.

- ***a***, ***b***, and ***c*** are called *parameters* or *coefficients*
- ***b*** and ***c*** can be any real number
- ***a*** can be any real number except 0

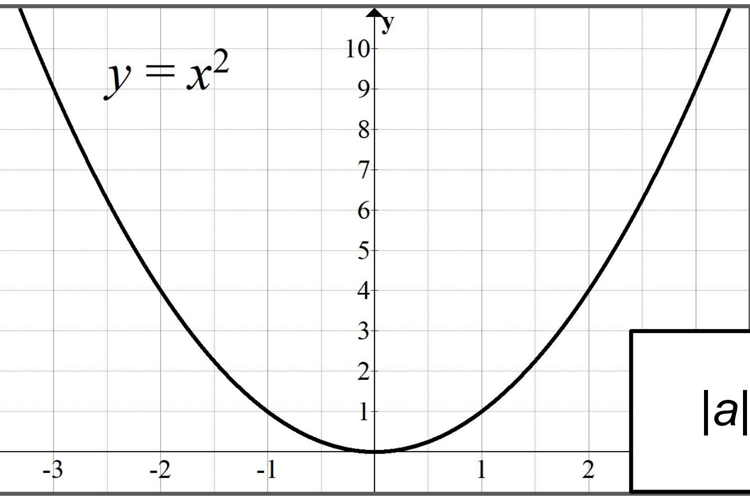
The simplest quadratic can be thought of as $\mathbf{y = x^2}$
which can be written as $\mathbf{y = 1x^2 + 0x + 0}$,
where $\mathbf{a = 1}$, $\mathbf{b = 0}$, **and** $\mathbf{c = 0}$.

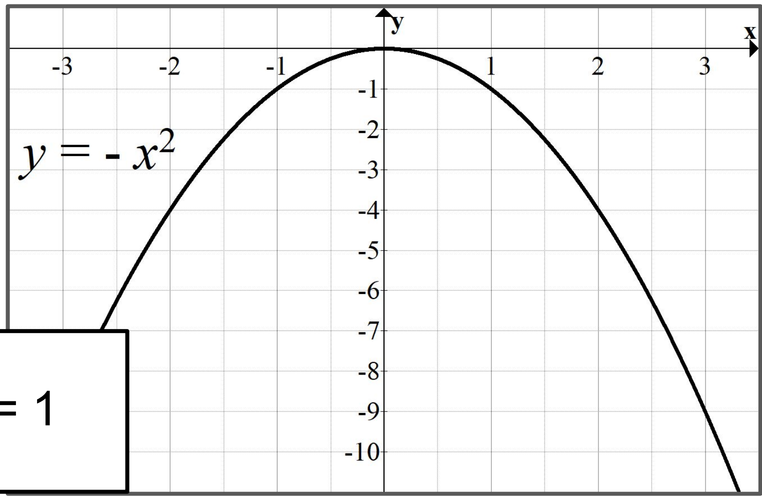
The graphical representation is



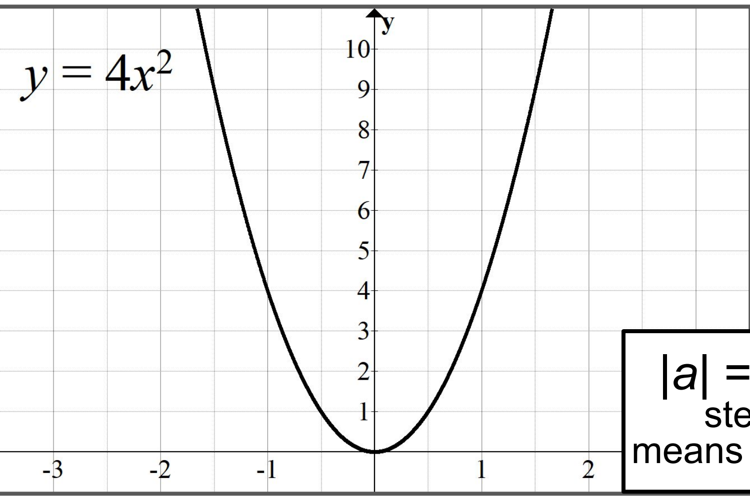
Unit 5 Introduction to the Quadratic Function Reference Sheet

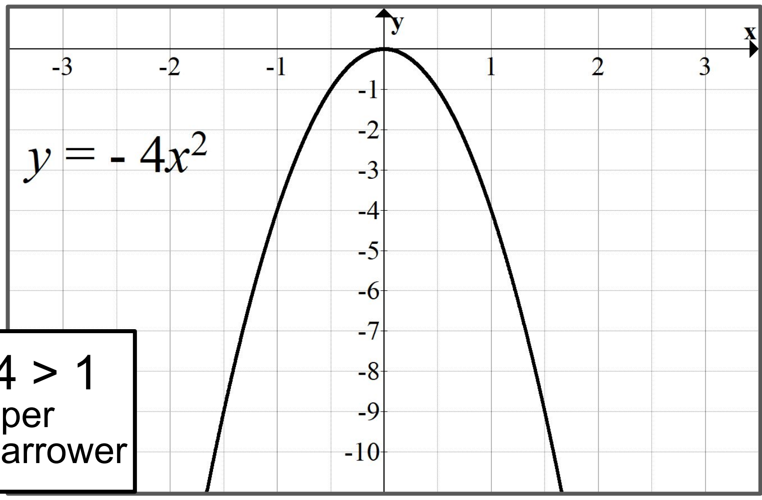
a is positive | *a* is negative



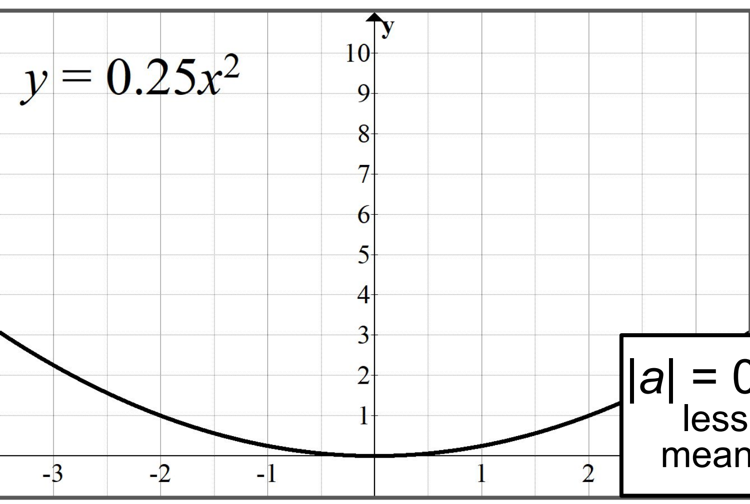


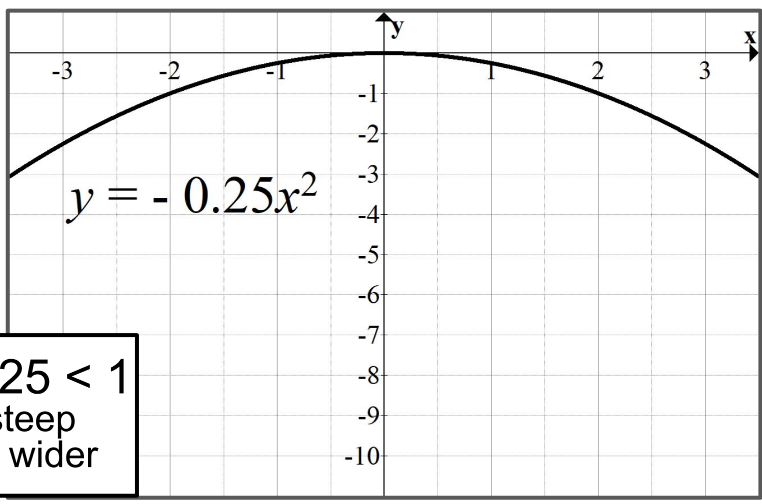
$|a| = 1$





$|a| = 4 > 1$
steeper
means narrower





$|a| = 0.25 < 1$
less steep
means wider