Unit: 5 - Solving Quadratic Equations Algebra I

Monday, March 14, 2016, 10:04AM

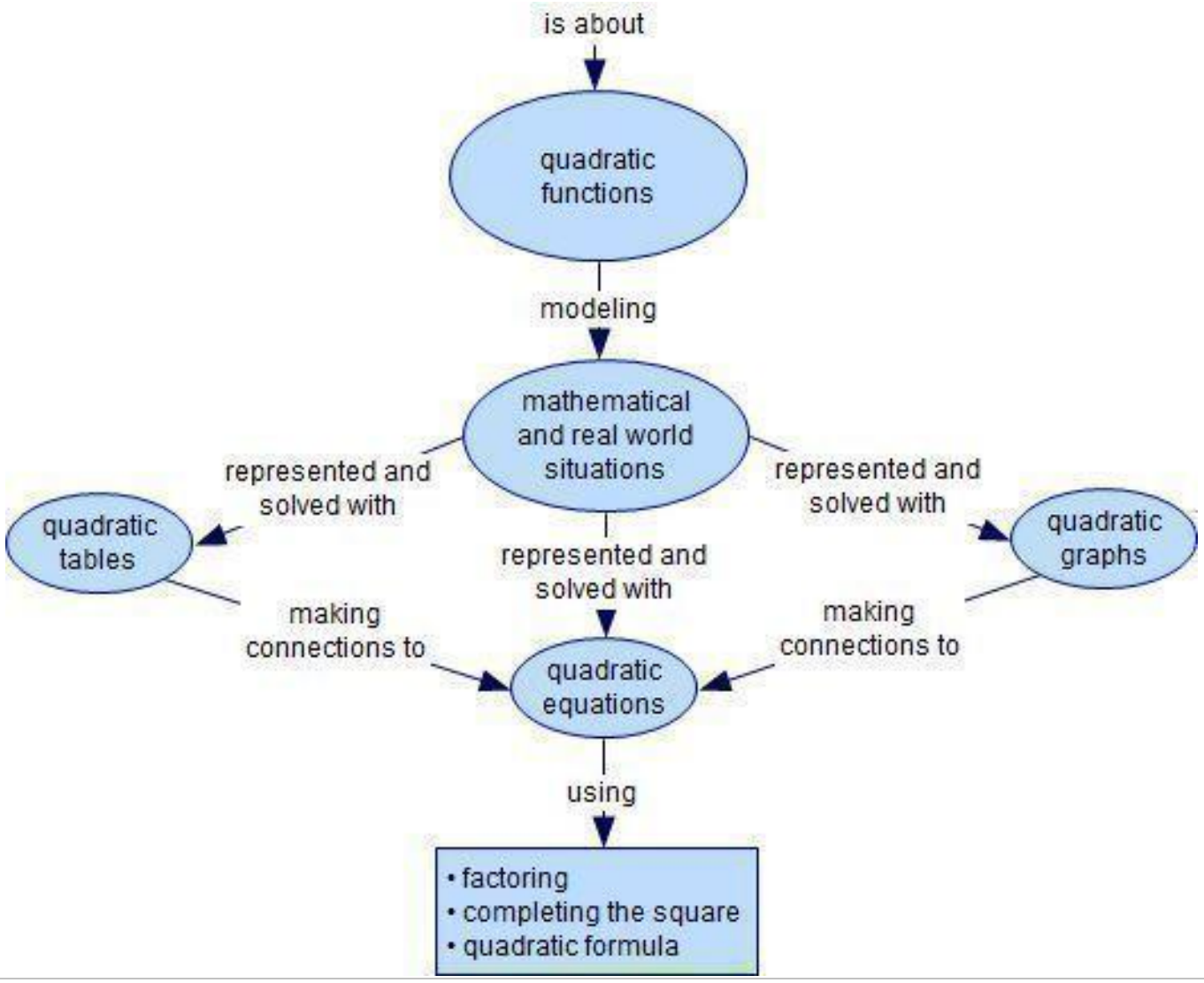
Common Core > 2015-2016 > Grade 9 > Mathematics > Algebra I (**CC**) > Week 24 - Week 29

Common Core Initiative

Overarching Questions and Enduring Understandings

What processes and representations can be used to find and interpret solutions of quadratic equations?

Graphic Organizer



Unit Abstract

In the last unit, students used multiple representations to make sense of quadratic functions and solve problems with tables and graphs. In this unit, students solve quadratic functions using what they learned from the last unit and begin to use algebraic strategies to solve quadratic equations. Students should identify which of these strategies is most efficient for any given situation. Additionally, when using algebraic strategies, students will produce real and imaginary/complex solutions. However, a more formal study of operations with imaginary/complex numbers will be

done in Algebra 2.
As students use algebraic strategies to find real or imaginary/complex solutions to quadratic equations, they can use what they know about transforming quadratic functions to see if their solutions makes sense. Similarly, students can use what they know about key features of quadratics (e.g., symmetry) to analyze or justify their answers.
The use of algebra tiles or area models provide opportunities for students to generate and make sense of algebraic strategies like completing the square and factoring. For example, as students repeatedly convert quadratics in standard form (represented with tiles) to vertex form, they see a pattern from the concrete model that leads to the procedure for completing the square. Once students understand the concept of completing the square, they can use it to make sense of the quadratic formula. (See an example of this process by using the following link.
https://www.youtube.com/watch?v=tHhO1_Snpsw)
In unit 2, students solved systems of linear equations both approximately, with tables and graphs, and exactly using algebraic strategies. In unit 3, students found approximate solutions for systems of equations consisting of both linear and exponential equations. In this unit, students extend those experiences to again find both approximate and exact solutions for systems that include linear, exponential and quadratic equations.

Content Expectations/Standards

High School: Number/Quantity

The Complex Number System

HSN-CN.C. Use complex numbers in polynomial identities and equations.

- HSN-CN.C.7. Solve quadratic equations with real coefficients that have complex solutions.

High School: Algebra

Seeing Structure in Expressions

HSA-SSE.B. Write expressions in equivalent forms to solve problems.

- HSA-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - HSA-SSE.B.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - HSA-SSE.B.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Arithmetic with Polynomials & Rational Functions

HSA-APR.B. Understand the relationship between zeros and factors of polynomials.

- HSA-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the CCSS-M standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

The Complex Number System

HSN-CN.A. Perform arithmetic operations with complex numbers.

- HSN-CN.A.1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations and Inequalities

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational,absolute value, exponential, and [quadratic] logarithmic functions.

Developed and/or Utilized Across Multiple Units

solving equations.

Reasoning with Equations & Inequalities

HSA-REI.A. Understand solving equations as a process of reasoning and explain the reasoning.

- HSA-REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

HSA-REI.B. Solve equations and inequalities in one variable.

- HSA-REI.B.4. Solve quadratic equations in one variable.
  - HSA-REI.B.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - HSA-REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

HSA-REI.C. Solve systems of equations.

- HSA-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.

High School: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

- HSF-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - HSF-IF.C.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Seeing Structure in Expressions

HSA-SSE.A. Interpret the structure of expressions.

- HSA-SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Reasoning with Equations & Inequalities

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Building Functions

HSF-BF.A. Build a function that models a relationship between two quantities.

- **HSF-BF.A.1.** Write a function that describes a relationship between two quantities.
  - **HSF-BF.A.1a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.

Essential/Focus Questions

1. What is the relationship between the number of real roots and the graph of a quadratic equation? Why does this relationship exists?
2. How does paying attention to the parameters of a quadratic function and its graph assist in finding solutions?
3. What are strategies for solving quadratic equations? When might one strategy be more efficient than another?
4. How is solving a system consisting of a linear or a non linear equation and a quadratic equation similar/different to solving linear systems and/or quadratic equations?

Key Concepts

algebraic strategies for solving (completing the square, factoring, quadratic formula)
discriminant
forms of quadratic functions (factored, standard, vertex)
imaginary/complex numbers
key features of quadratics graphs (vertex, axis of symmetry, minimum, maximum, x and y intercepts, end behavior)
models of quadratic functions (tables, graphs, equations, algebra tiles)
quadratic patterns (recursive, common second difference, explicit)
solutions (real or imaginary/complex roots, zeroes, x-intercepts)
solutions to non-linear systems

Assessment Tasks

Assessment Overview
Student Handout Reposted with permission
Teacher Notes

Intellectual Processes

Standards for Mathematical Practice

Students will have opportunities to:

- **look for and make use of structure** through the parameters and the features of the graph to find solutions to quadratic equations;
- **look for and express regularity in repeated reasoning** when using the quadratic formula to identify real or imaginary/complex roots of a quadratic equation; and
- **model with mathematics** by using algebra tiles to make sense of the procedures like completing the square and factoring.

Lesson Sequence

Lesson Overview
Student Handout
Answer Key

Resources

Unit Resources

Atlas Version 8.1.1