

CONNECTIONS: Michigan Academic State Standards for Mathematics

EXAMPLE CONTEXT FOR LANGUAGE USAGE: The purpose of this activity is for students to notice changes in the key features of graphs when $f(x)$ is replaced with $f(x) + k$, $k f(x)$, $f(kx)$, or $f(x + k)$. To begin the lesson, teachers should engage students in dialogue of the key features of polynomial functions (y-intercept, end behavior, local maximum/minimum, maximum/minimum), and transformations of functions (horizontal/vertical translation, reflection, horizontal/vertical dilation). This dialogue serves both as a review and a formative assessment opportunity for the teacher and students to simultaneously check understanding of the mathematics and language of key features. Listening and speaking in this dialogue provides a scaffold for the rest of the task where students will explore multiple functions.

The strand below illustrates another listening demand of the lesson where students will listen to the directions for one particular problem that serves as an example for the other collection of polynomials that they will be investigating. (This process will be repeated multiple times with different functions and/or changes. Students at all levels of proficiency will be encouraged to compare work with a partner. Students should make predictions about changes and compare with a partner using technology to verify.

At the end of the investigation, students will listen and speak again as the class collectively summarizes changes in the graph based on changes in the algebraic representation ($f(x)$ is replaced with $f(x) + k$, $k f(x)$, $f(kx)$, or $f(x + k)$).

COGNITIVE FUNCTION: Students at all levels of English language proficiency will **SYNTHESIZE** instructions in order to investigate and summarize how changes in values of the parameters in a polynomial function affect the key attributes of the graph (i.e., y-intercept, end behavior, maximum/minimum) and related transformations (horizontal/vertical translation, reflection, horizontal/vertical dilation).

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Synthesize instructions read aloud multiple times (with purposeful pauses and pointing to the appropriate part of the activity sheet) for writing a polynomial function and its transformation, sketch graphs of both the original and transformed functions showing the key attributes (y-intercept, end behavior, horizontal/vertical translation, reflection, horizontal/vertical dilation) and then verify the answer by using technology to view the graph, referring to a related completed model and working with a partner at a higher level of English language proficiency.	Synthesize instructions read aloud multiple times (with purposeful pauses and pointing to the appropriate part of the activity sheet) for writing a polynomial function and its transformation, sketch graphs of both the original and transformed functions showing the key attributes (y-intercept, end behavior, horizontal/vertical translation, reflection, horizontal/vertical dilation) and then verify the answer by using technology to view the graph, referring to a related completed model and working with a partner.	Synthesize instructions read aloud multiple times (with purposeful pauses) for writing a polynomial function and its transformation, sketch graphs of both the original and transformed functions showing the key attributes (y-intercept, end behavior, horizontal/vertical translation, reflection, horizontal/vertical dilation) and then verify the answer by using technology to view the graph, while working with a partner.	Synthesize instructions read aloud (with purposeful pauses) for writing a polynomial function and its transformation, sketch graphs of both the original and transformed functions showing the key attributes (y-intercept, end behavior, horizontal/vertical translation, reflection, horizontal/vertical dilation) and then verify the answer by using technology to view the graph, comparing work with a partner.	Synthesize instructions read aloud (with purposeful pauses) for writing a polynomial function and its transformation, sketch graphs of both the original and transformed functions showing the key attributes (y-intercept, end behavior, horizontal/vertical translation, reflection, horizontal/vertical dilation) and then verify the answer by using technology to view the graph, comparing work with a partner.	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	<p>E.g., "[The teacher writes the original function and points to it as a reference throughout the task.] Write the function [pause and point] f of x equals [pause and point] x cubed [pause and point] minus x squared [pause and point] plus 5 [pause and point], and create an accurate graph on your activity sheet [pause and point to the part of the activity sheet where students should create the graph].</p> <p>Now, write the function [pause and point to the part of the activity sheet where students should write the new function] f of the quantity $(x+3)$ [pause and scribe as needed to clarify].</p> <p>On the same coordinate plane [pause and point], sketch a graph that you believe will best represent the new function [pause while students sketch the graph]. Compare your prediction with a partner [pause]. Now use technology to verify your prediction."</p>	<p>E.g., "[Point to the appropriate part of the activity sheet.] Write the function [pause] f of x equals [pause] x cubed [pause] minus x squared [pause] plus 5 [pause], and create an accurate graph on your activity sheet [pause and point].</p> <p>Now, write the function [pause and point] f of the quantity $(x+3)$ [pause and scribe as needed to clarify].</p> <p>On the same coordinate plane [pause and point], sketch a graph that you believe will best represent the new function [pause while students sketch the graph]. Compare your prediction with a partner [pause]. Now use technology to verify your prediction."</p>	<p>E.g., "Write the function [pause] f of x equals [pause] x cubed [pause] minus x squared [pause] plus 5 [pause], and create an accurate graph on your activity sheet [pause while students create the graph].</p> <p>Now, write the function [pause] f of the quantity $(x+3)$ [pause and scribe as needed to clarify].</p> <p>On the same coordinate plane [pause], sketch a graph that you believe will best represent the new function [pause while students sketch the graph].</p> <p>Compare your prediction with a partner [pause]. Now use technology to verify your prediction."</p> <p>*sample activity sheet provided in supports</p>	<p>E.g., "Write the function [pause] f of x equals [pause] x cubed [pause] minus x squared [pause] plus 5 [pause], and create an accurate graph on your activity sheet [pause while students create the graph].</p> <p>Now, write the function [pause] f of the quantity $(x+3)$ [pause and scribe as needed to clarify].</p> <p>On the same coordinate plane [pause], sketch a graph that you believe will best represent the new function [pause while students sketch the graph].</p> <p>Compare your prediction with a partner [pause]. Now use technology to verify your prediction."</p>	<p>E.g., "Write the function [pause] f of x equals [pause] x cubed [pause] minus x squared [pause] plus 5 [pause], and create an accurate graph on your activity sheet [pause while students create the graph].</p> <p>Now, write the function [pause] f of the quantity $(x+3)$ [pause and scribe as needed to clarify].</p> <p>On the same coordinate plane [pause], sketch a graph that you believe will best represent the new function [pause while students sketch the graph].</p> <p>Compare your prediction with a partner [pause]. Now use technology to verify your prediction."</p>	

EXAMPLE CONTEXT FOR LANGUAGE USAGE: This lesson addresses HSF-IF.C.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. The lesson below is an adaptation of a polynomial lesson found at <http://map.mathshell.org/lessons.php?unit=9270&collection=8>, whose main focus is on cubic polynomials. Please read through the entire mathshell lesson for a complete version and suggested implementation of the following parts: before the lesson, unit lesson, lesson extension, and after the lesson. The adaptation shown below focuses on the unit lesson itself (modified to include higher degree polynomials) where students work collaboratively in pairs, matching functions to their graphs and creating new examples when needed. Throughout their work, students justify and explain their decisions to peers. During a whole-class discussion, students explain their reasoning.

Launch the lesson with the following instructions:

1. Take turns to match a function to its graph.
2. As you do this, label the graph to show the intercepts on the x- and y-axes.
3. If you match two cards, explain how you came to your decision.
4. If you don't agree or understand, ask your partner to explain their reasoning.
5. You both need to agree on and be able to explain the matching of every card.
6. You may find that more than one function will match some graphs.
7. If you have functions left over, sketch graphs on the blank cards to match these functions.

Note: Students do not need to use the same strategy as described in the sample response below. As students are working, try not to make suggestions that move students toward a particular strategy. Instead, ask questions to help students to reason together. Encourage students to use an efficient method.

English Learners at WIDA Levels 1 and 2 are paired with a partner of higher language proficiency so, as they take turns explaining, the student with higher proficiency models the vocabulary which helps reinforce the connections. The teacher could strategically choose a partner higher in mathematical understanding as well as language proficiency, encouraging the student with higher proficiency to point during the explanation as well.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **EXPLAIN** to a partner the strategy used to match a polynomial function to its graph.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking	Explain in words and/or short phrases, the strategy used to match a polynomial function to its graph using the unit anchor chart while gesturing or pointing to the anchor chart and/or function/graph on cards and working with a partner of higher English language proficiency.	Explain in simple sentences the strategy used to match a polynomial function to its graph using the unit anchor chart and sentence frames with choices while gesturing or pointing to the anchor chart and/or function/graph on cards and working with a partner of higher English language proficiency.	Explain in complete sentences the strategy used to match a polynomial function to its graph using the unit anchor chart and sentence frames with choices while working with a partner of similar or higher English language proficiency. [Note: Students may use one or more of the following sentence frames.]	Explain using compound and/or complex sentences the strategy used to match a polynomial function to its graph using the unit anchor chart and suggested word list (e.g., zeros, roots, x-intercepts, double roots, end behavior, function, odd degree, even degree, leading coefficient, graph, x-axis, y-axis).	Explain using compound and/or complex sentences the strategy used to match a polynomial function to its graph using the unit anchor chart.	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	<p>E.g., "x-intercepts" [pointing to the x-intercepts on the graph]</p> <p>"Factors" [pointing to the factors of the polynomial function] "Match/Same" [indicating that they go together]"Degree ____" [pointing to exponents]"Opposite or Same" [pointing to the end behavior on the graph or gesturing opposite/same direction]</p> <p>"Positive or Negative" [pointing to the leading coefficient of the function]</p> <p>"Up or Down" [indicating the direction of each end of the graph or gesturing to show what the end behavior should be]</p>	<p>[Note: Students may use one or more of the following sentence frames.]</p> <p>The _____ (graph/function) has _____ (zeros/x-intercepts/factors) at/of _____.</p> <p>The _____ (graph/function) must have _____ (zeros/x-intercepts/factors) at/of _____.</p> <p>A _____ (single/double/triple/...) root would cause the graph to _____ (go through/just touch) the x-axis.</p> <p>The degree of the function is _____ (odd/even).</p> <p>The end behavior of the graph must start and end in _____ (same/opposite) direction.</p> <p>The leading coefficient is _____ (positive/negative) so the end behavior must start _____ (up/down) and end _____ (up/down).</p> <p>[repeated for each matching pair]</p>	<p>The _____ (graph/function) has _____ (zeros/x-intercepts/factors) at/of _____.</p> <p>So, the _____ (graph/function) must have _____ (zeros/x-intercepts/factors) at/of _____.</p> <p>A _____ (single/double/triple/...) root would cause the graph to _____ (go through/just touch) the x-axis.</p> <p>The degree of the function is _____ (odd/even), so the intercepts/factors) at/of _____.</p> <p>end behavior of the graph must start and end in the _____ (same/opposite) direction.</p> <p>The leading coefficient is _____ (positive/negative), so the end behavior must start _____ (up/down) and end _____ (up/down).</p> <p>[repeated for each matching pair]</p>	<p>E.g., "Looking at the function $f(x)=(x+3)^2(x+1)(x-1)(x-5)$, I noticed that the zeros of the function were -3 (double root) -1, 1, and 5 which told me the graph needed to have x-intercepts at these values. Since $x = -3$ was a double root, the graph had to touch the x-axis here but not pass through it. Given that the function has a degree of 5 (odd), the end behavior of the graph needed to be in opposite directions. Since the leading coefficient is positive, the graph needed to start down and end up."</p>	<p>E.g., "Looking at the function $f(x)=(x+3)^2(x+1)(x-1)(x-5)$, I noticed that the zeros of the function were -3 (double root) -1, 1, and 5 which told me the graph needed to have x-intercepts at these values. Since $x = -3$ was a double root, the graph had to touch the x-axis here but not pass through it. Given that the function has a degree of 5 (odd), the end behavior of the graph needed to be in opposite directions. Since the leading coefficient is positive, the graph needed to start down and end up."</p>	

ELD STANDARD 3: The Language of Mathematics

MAISA Algebra II, Unit 1, Power and Polynomial Functions

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will analyze a linguistically complex piece of the mathematical text and apply this knowledge to additional examples. It is important that students read for a purpose of understanding the mathematics, but there would also need to be some form of language production (i.e., writing or speaking). The strand below illustrates just the reading and is taken from Chapter 8 of the College Preparatory Mathematics (CPM) Core Connections Algebra 2 book: Kysh, J., Dietiker, L., Sallee, T., & Hoey, B. (2013). Core connections: Algebra 2. Sacramento, CA: CPM Educational Program.

Note that in the strand of differentiation written below, the language function is to analyze a linguistically complex piece of mathematical text, but the text would be scaffolded for students at Levels 1 and 2 as shown in the supports. At Level 3, students would produce a similar version of what the teacher provided at Levels 1 and 2. Alternatively, a teacher might provide a simplified version of the text or change the original version by integrating visual aids.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** a linguistically complex piece of the mathematical text in order to **IDENTIFY** key features of given polynomial functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Analyze a linguistically complex piece of mathematical text that has been color coded in order to identify key features of given polynomial functions, while working with a partner of higher proficiency in mathematical understanding and language.	Analyze a linguistically complex piece of mathematical text that has been color coded in order to identify key features of given polynomial functions, while working with a partner of higher proficiency in mathematical understanding and language.	Analyze a linguistically complex piece of mathematical text by color coding in order to identify key features of given polynomial functions, while working with a group of students with similar or higher language proficiency.	Analyze a linguistically complex piece of mathematical text in order to identify key features of given polynomial functions, checking work with a partner.	Analyze a linguistically complex piece of mathematical text in order to identify key features of given polynomial functions, checking work with a partner.	

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will identify the zeros of a polynomial function (in factored or expanded, yet factorable, form) and explain the relationships among the zeros, factors, and x-intercepts of the function and its graph. Students may construct multiple representations, either by hand or using technology, to support sense-making. Depending on the writing proficiency level of the Level 2 student, the teacher may choose to provide scaffolds similar to those for either Level 1 or Level 3 and assigned a partner accordingly. NOTE: the required word list is shortened at Level 1 so that students would know how to label their work; because there are so few words on the word list, it was made required, not suggested.

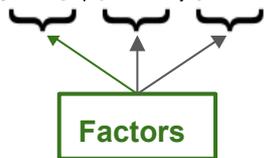
COGNITIVE FUNCTION: Students at all levels of English language proficiency **EXPLAIN** the relationships among the zeros, factors, and x-intercepts of a polynomial function and its graph.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Writing	<p>Explain by labeling work in words, short phrases, and/or simple sentences, the relationships among the zeros, factors, and x-intercepts of a polynomial and its graph using a shortened required word list (e.g., x-intercept(s), zero(s), factor(s)), an anchor chart and illustrated reference sheet while working with a partner of a similar proficiency level.</p> <p>[See the example of student work in the supports.]</p>	<p>Explain in complete sentences the relationships among the zeros, factors, and x-intercepts of a polynomial and its graph using a suggested word list (e.g., x-intercept(s), point, graph, zero(s), factor(s), solution(s), x-axis, polynomial, equal), an anchor chart, an illustrated reference sheet, while working with a strategically assigned partner.</p> <p>E.g., "In the function $f(x) = 2x(x-3)(x+4)$, the factors are $2x$, $(x-3)$, and $(x+4)$. The zeros are $x=0$, $x=3$ and $x=-4$. The zeros are the solutions when the equation is set equal to zero. I found the zeros by setting the factors equal to zero and solving. The graph hits the x-axis when the y-value is zero, so the x-intercepts are $(0,0)$, $(3,0)$ and $(-4,0)$."</p>	<p>Explain in complete sentences the relationships among the zeros, factors, and x-intercepts of a polynomial and its graph using a suggested word list (e.g., x-intercept(s), point, graph, zero(s), factor(s), solution(s), x-axis, polynomial, equal), an anchor chart, an illustrated reference sheet, while working with a partner.</p> <p>E.g., "In the function $f(x) = 2x(x-3)(x+4)$, the factors are $2x$, $(x-3)$, and $(x+4)$. The zeros are $x=0$, $x=3$ and $x=-4$. The zeros are the solutions when the equation is set equal to zero. I found the zeros by setting the factors equal to zero and solving. The graph hits the x-axis when the y-value is zero, so the x-intercepts are $(0,0)$, $(3,0)$ and $(-4,0)$."</p>	<p>Explain in compound and/or complex sentences the relationships among the zeros, factors, and x-intercepts of a polynomial and its graph using a suggested word list (e.g., x-intercept(s), point, graph, zero(s), factor(s), solution(s), x-axis, polynomial, equal), an anchor chart and working with a partner.</p> <p>E.g., "In the function $f(x) = 2x(x-3)(x+4)$, the factors are $2x$, $(x-3)$, and $(x+4)$, so the zeros are $x=0$, $x=3$ and $x=-4$. The x-intercepts are $(0,0)$, $(3,0)$ and $(-4,0)$. The x-intercepts are the points where the graph touches or crosses the x-axis, thus they are the x-values when the y-value is zero. Zeros refer to the solutions to the equation when it is set equal to zero. When we set the factors equal to zero and solve, we find the x-values that make the polynomial equal to zero,</p>	<p>Explain in compound and/or complex sentences the relationships among the zeros, factors, and x-intercepts of a polynomial and its graph using a required word list (e.g., x-intercept(s), point, graph, zero(s), factor(s), solution(s), x-axis, polynomial, equal), an anchor chart and working with a partner.</p> <p>E.g., "In the function $f(x) = 2x(x-3)(x+4)$, the factors are $2x$, $(x-3)$, and $(x+4)$, so the zeros are $x=0$, $x=3$ and $x=-4$. The x-intercepts are $(0,0)$, $(3,0)$ and $(-4,0)$. The x-intercepts are the points where the graph touches or crosses the x-axis, thus they are the x-values when the y-value is zero. Zeros refer to the solutions to the equation when it is set equal to zero. When we set the factors equal to zero and solve, we find the x-values that make the</p>	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued				which are the x-intercepts of the graph and the zeros of the function."	polynomial equal to zero, which are the x-intercepts of the graph and the zeros of the function."	

Example of polynomial function in factored form:

$$f(x) = (x - p)(x - n)(x - r)$$



Finding zeros of polynomial function in factored form:

$$0 = (x - p)(x - n)(x - r)$$

$$x - p = 0 \quad \text{OR} \quad x - n = 0 \quad \text{OR} \quad x - r = 0$$

$$x = p$$

$$x = n$$

$$x = r$$

Zeros

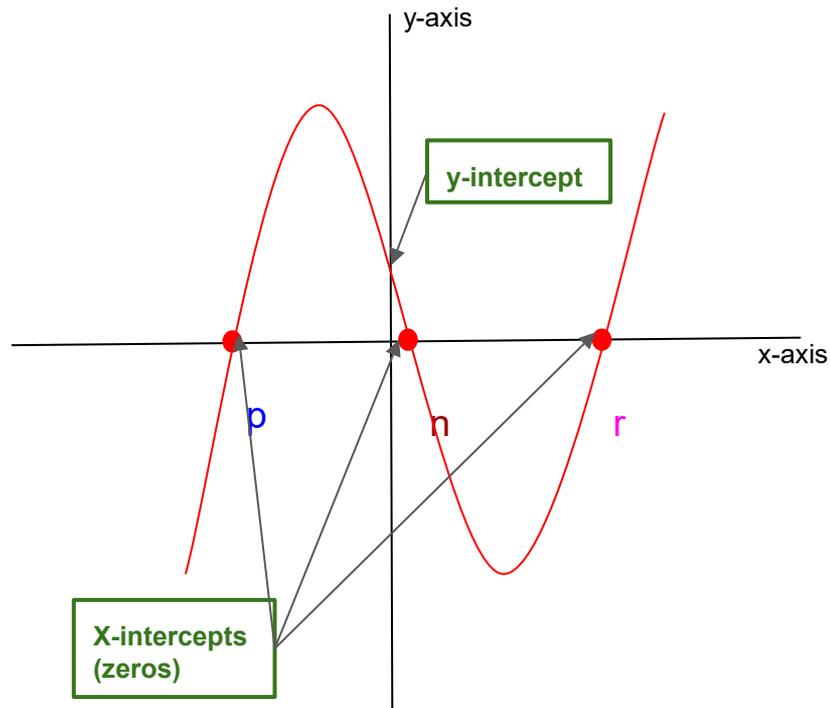
$$(p, 0)$$

$$(n, 0)$$

(x - intercepts)

$$\left. \begin{aligned} f(p) &= (p - p)(p - n)(p - r) = 0 \cdot (p - n)(p - r) = 0 \\ f(n) &= (n - p)(n - n)(n - r) = (n - p) \cdot 0 \cdot (n - r) = 0 \\ f(r) &= (r - p)(r - n)(r - r) = (r - p)(r - n) \cdot 0 = 0 \end{aligned} \right\}$$

When you evaluate a function at its zero, the value is ZERO!



Examples of Polynomial Functions

General form:

$$g(x) = -3x^6 + 4x^2 - 6x + 7$$

Leading coefficient

Degree of polynomial

Factored form:

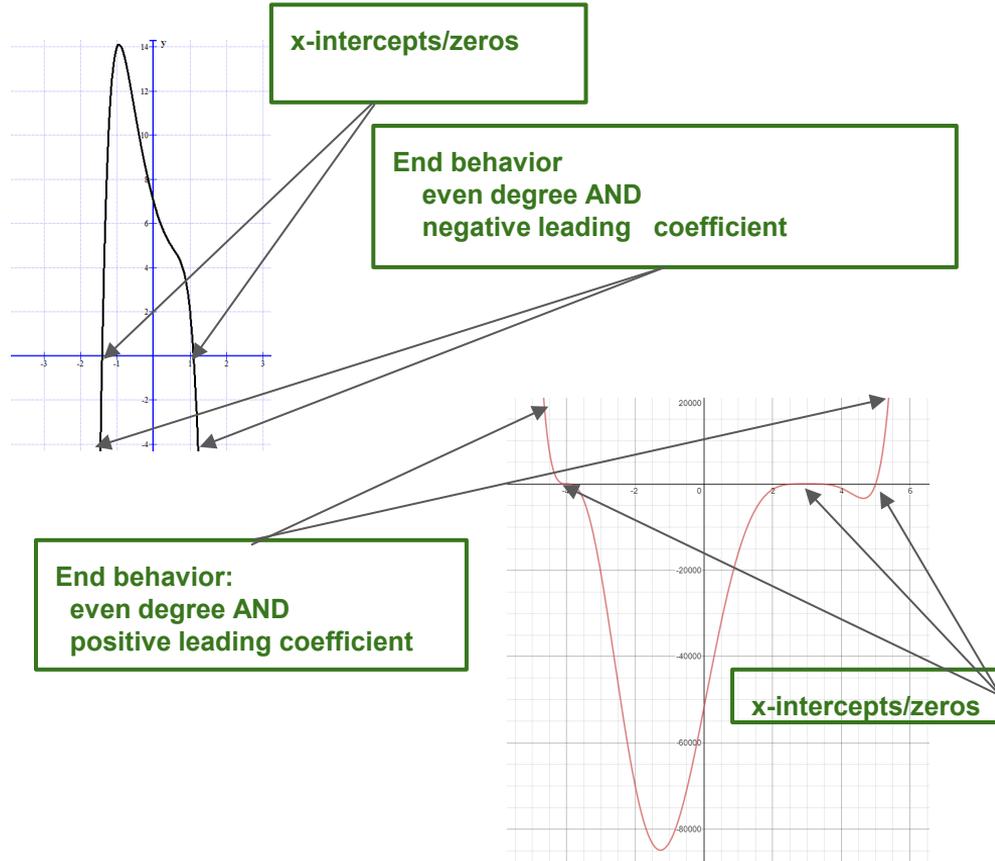
$$h(x) = 2(x - 1)^4(x + 2)^3(x - 5)^1$$

Leading coefficient

8

$$4 + 3 + 1 =$$

Degree of polynomial

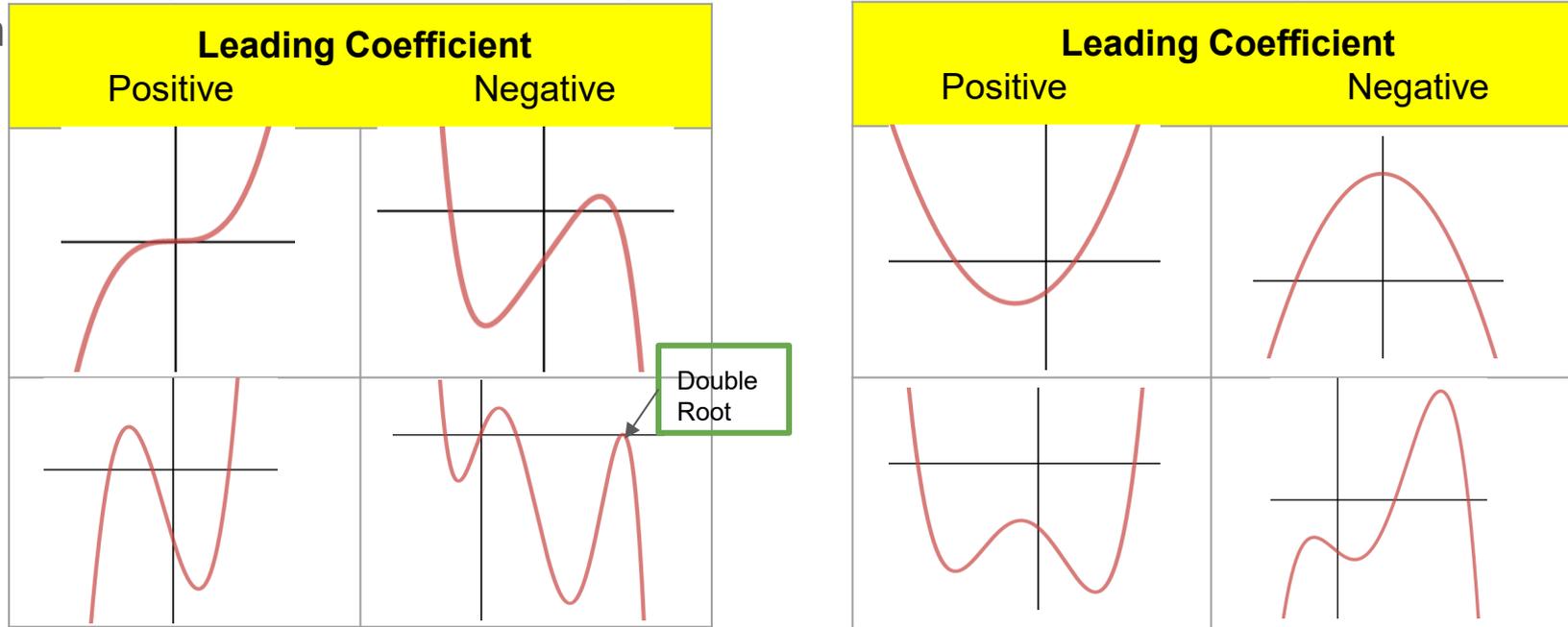


End Behavior of

Polynomials with Odd Degree

Polynomials with

Even



The end behavior of polynomials with _____ (odd/even) degree
and a _____ (positive/negative) leading coefficient is...

Tools for Finding Zeros, Roots, x-intercepts

Example: $f(x) = x^3 - 13x + 12 = (x-3)(x-1)(x+4)$

Algebraically/Symbolically

- Use the Rational Roots Theorem

p: ± 1

q: $\pm 1, \pm 2, \pm 3, \pm 4$

Possible roots: $\pm p/q$

- Evaluate the Function

$$f(-4) = -4^3 - 13(-4) + 12 = 0$$

$$f(-3) = -3^3 - 13(-3) + 12 = 24$$

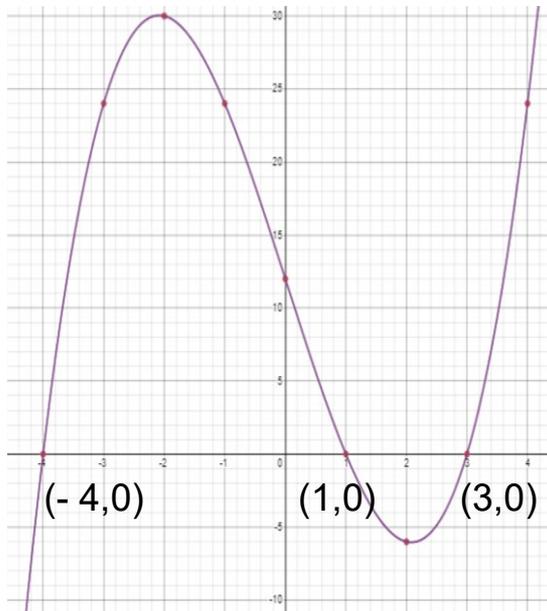
- Long Division by a factor

$$\begin{array}{r}
 x^2 - 4x + 3 \\
 x^3 - 0x^2 - 13x + 12 \\
 \hline
 x^3 + 4x^2 \\
 \hline
 -4x^2 - 13x + 12 \\
 -4x^2 - 16x \\
 \hline
 3x + 12 \\
 3x + 12 \\
 \hline
 0
 \end{array}$$

- Factor the polynomial

$$f(x) = (x-3)(x-1)(x+4)$$

Graphically



Numerically (on a Table)

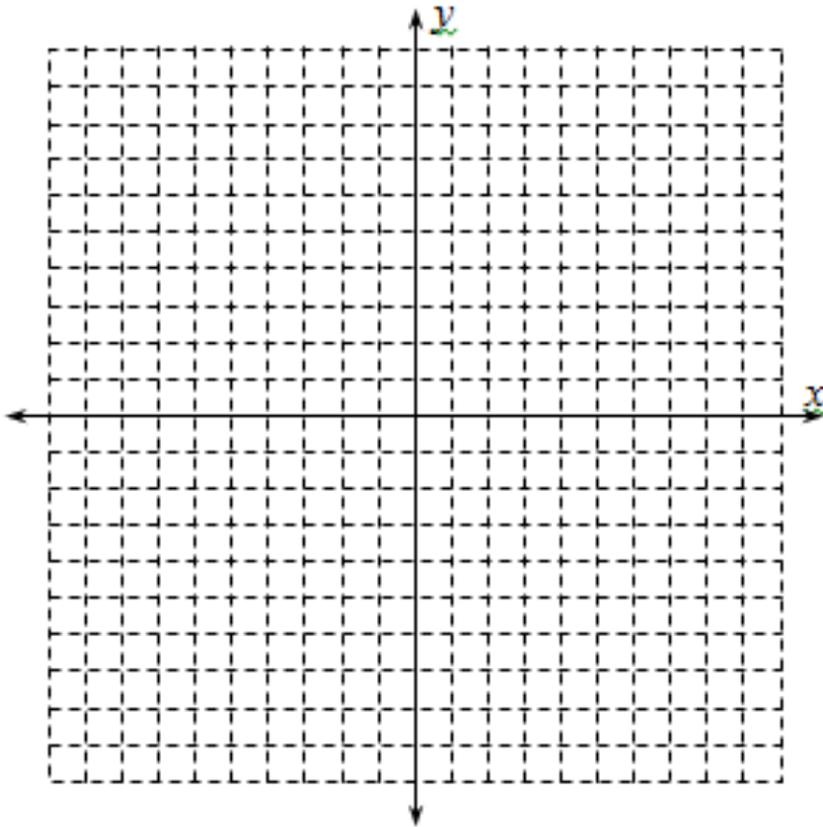
x	$f(x) = x^3 - 13x + 12$
-5	-48
-4	0
-3	24
-2	30
-1	24
0	12
1	0
2	-6
3	0
4	24

x- values where the function value is 0

Original Function: _____

New Function: _____

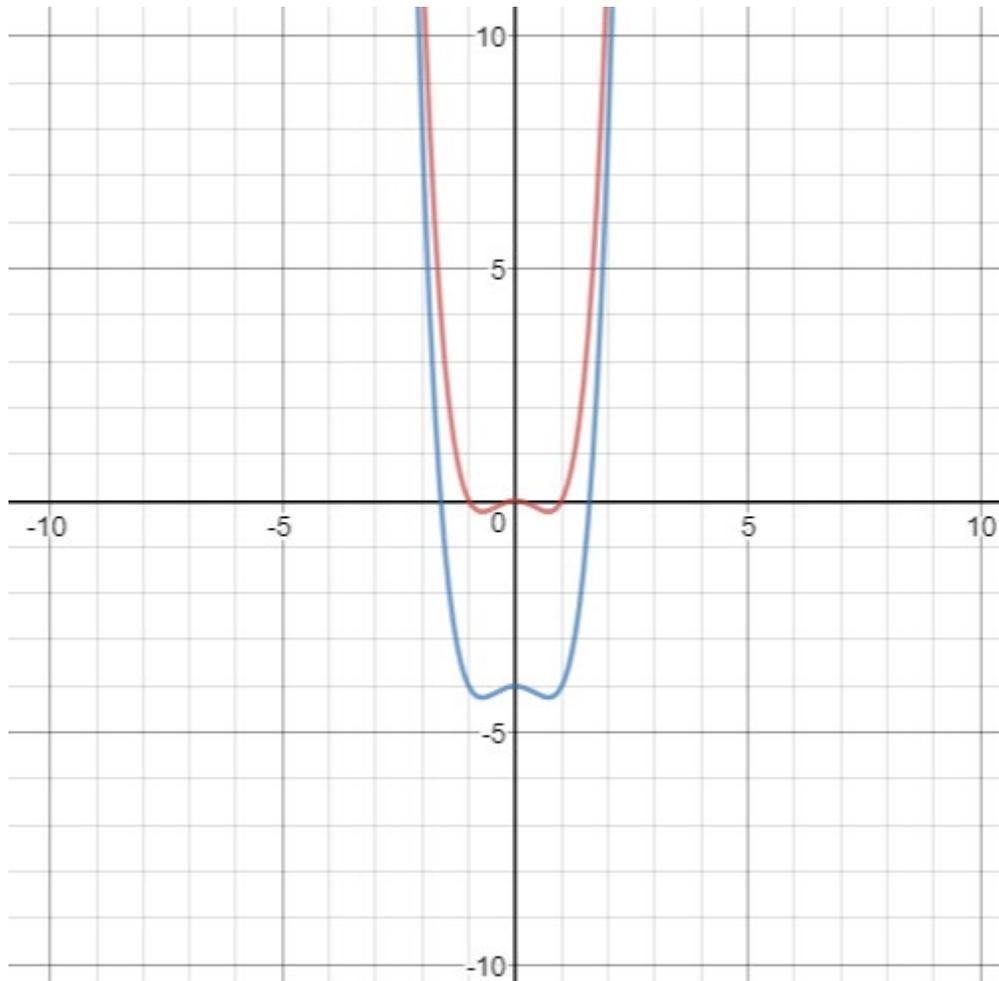
Graph:

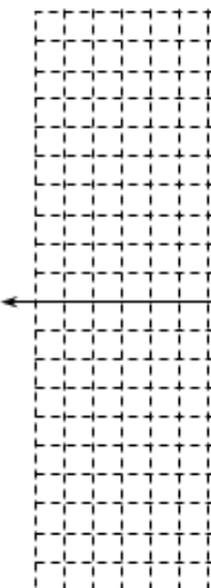


Original Function: $f(x) = x^4 - x^2$

New Function: $f(x) = (x^4 - x^2) - 4$

Graph:



<p>graph of $f(x)$ with equation</p>		<p>key feature of $f(x)$</p>	
<p>$f(x) + k,$</p>	<p>key features</p>	<p>$k f(x)$</p>	
<p>$f(x + k)$</p>		<p>$f(kx)$</p>	

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Matching Representations of Polynomials Functions

Notes for problem creation:

Three sample pairs of polynomials are included below to illustrate cards that can be created. While three cards might be sufficient for a minilesson, it is not likely to be an appropriate amount for a whole class activity. In actually creating the activity, consider the appropriate number of cards for students to both match and justify their thinking. There should be enough representations to provide opportunities for comparison, practice, and justification but not too many that the exercise is tedious or overwhelming. (For example, three might be too few but fifteen pairs might be overly time-consuming for students.)

Additionally, the cards teachers create also determine the amount of problem solving, productive struggle, and reasoning in which students are given opportunities to engage. Considerations to potentially increase the cognitive demand are listed below.

- Vary the algebraic/symbolic representation to include both factored or standard forms to allow multiple symbolic representations to fit one graphical or tabular representations.
- Include tabular representations.
- Create some cards that allow multiple valid responses.
 - For example, $f(x) = (x-1)^2 (x-2)^3$ vs. $g(x) = (x-1)^4 (x-2)^5$. These are two different functions, but based on degree, roots, and end behavior, it would be unclear whether a graphical representation represented f or g .
- Create cards that do not have matches provided. Along with these cards also provide empty cards on which students can create an appropriate match.

A

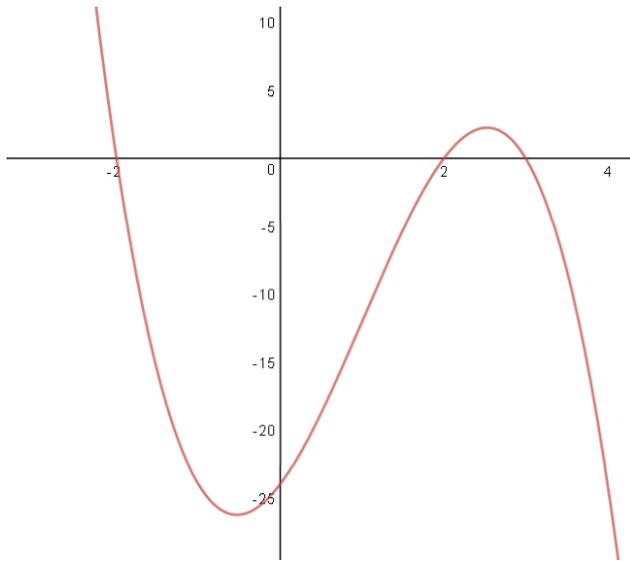
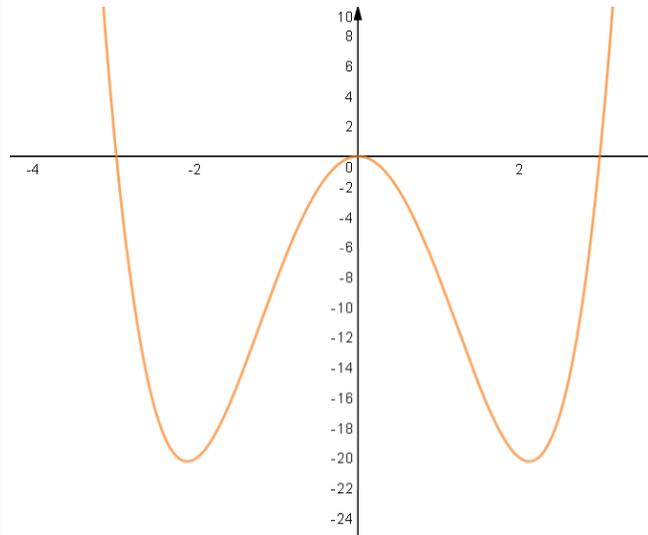
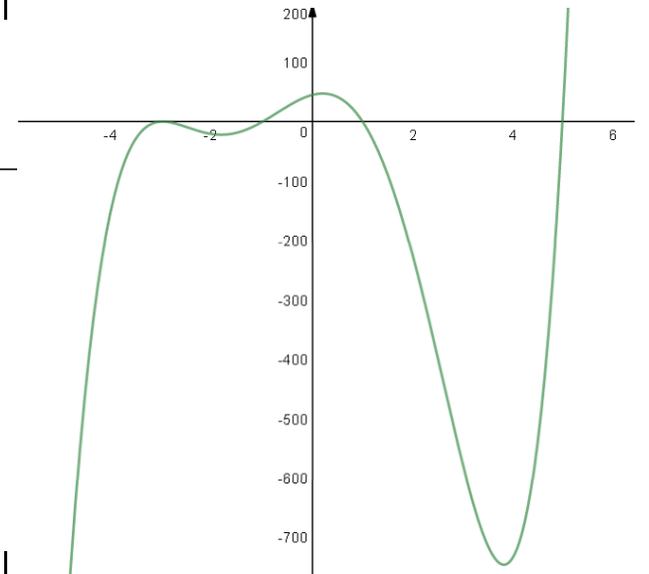
$$f(x) = (x + 3)^2(x + 1)(x - 1)(x - 5)$$

B

$$f(x) = -2(x - 2)(x + 2)(x - 3)$$

C

$$f(x) = x^4 - 9x^2$$

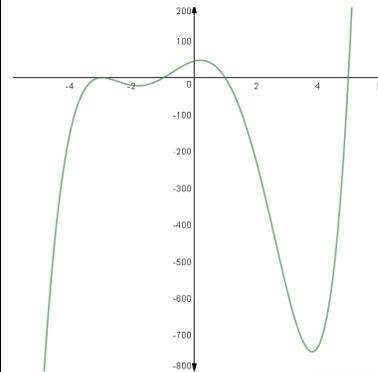
D**E****F**

Answers

Links to where the graphs were created are also provided in case teachers and students might like to use them in justification.

A. $f(x) = (x + 3)^2(x + 1)(x - 1)(x - 5)$

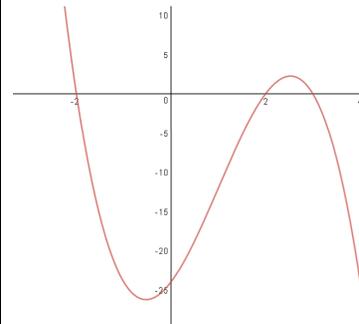
F.



<https://www.desmos.com/calculator/amwsijqzq>

B. $f(x) = -2(x - 2)(x + 2)(x - 3)$

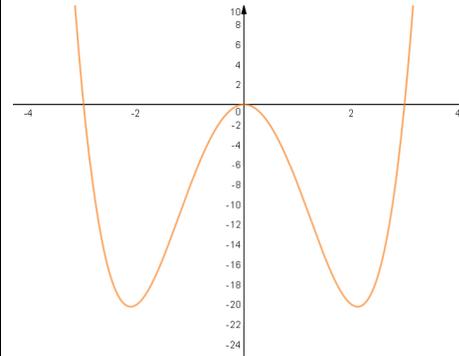
D.



<https://www.desmos.com/calculator/roklcvsgyc>

C. $f(x) = x^4 - 9x^2$

E.



<https://www.desmos.com/calculator/iqihxvkrp>



METHODS AND MEANINGS

Polynomials, Degree, Coefficients

Refer to the Math Notes box in Lesson 3.1.3 for an explanation of a **polynomial** in one variable.

Polynomials with one variable (often x) are usually arranged with powers of x in order, starting with the highest, left to right.

The highest power of the variable in a polynomial of one variable is called the **degree** of the polynomial. The numbers that multiply each term are called **coefficients**. See the examples below.

Example 1: $f(x) = 7x^5 + 2.5x^3 - \frac{1}{2}x + 7$ is a polynomial function of degree 5 with coefficients 7, 0, 2.5, 0, $-\frac{1}{2}$, and 7. Note that the last term, 7, is called the **constant term** but represents the variable expression $7x^0$, since $x^0 = 1$.

Example 2: $y = 2(x + 2)(x + 5)$ is a polynomial in factored form with degree 2 because it can be written in standard form as $y = 2x^2 + 14x + 20$. It has coefficients 2, 14, and 20.



MATH NOTES

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factors

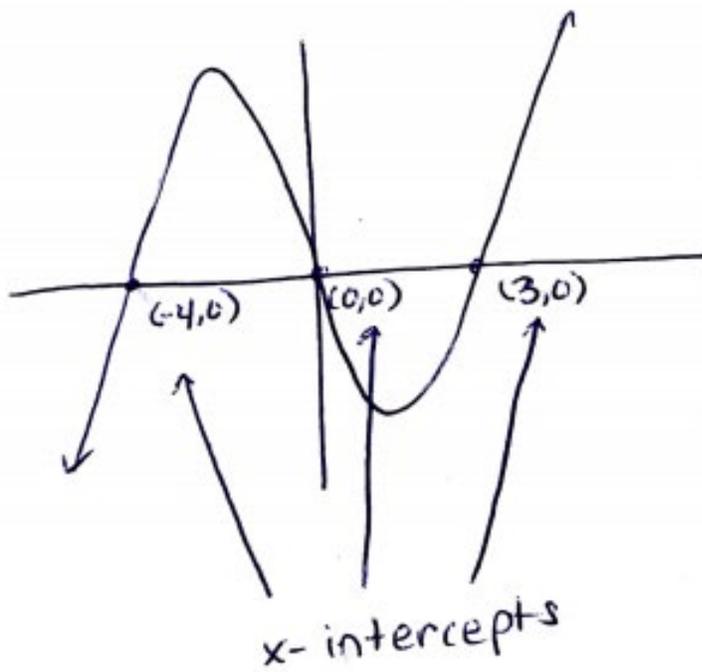
$$f(x) = 2x(x-3)(x+4)$$

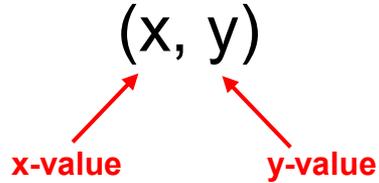
$\frac{2x=0}{\frac{2}{2}} \quad x-3=0 \quad x+4=0$
 $\quad \quad \quad +3 \quad +3 \quad -4 \quad -4$

$x=0 \quad x=3 \quad x=-4$

Zeros

Zeros come from factors.
Zeros give x-intercepts.



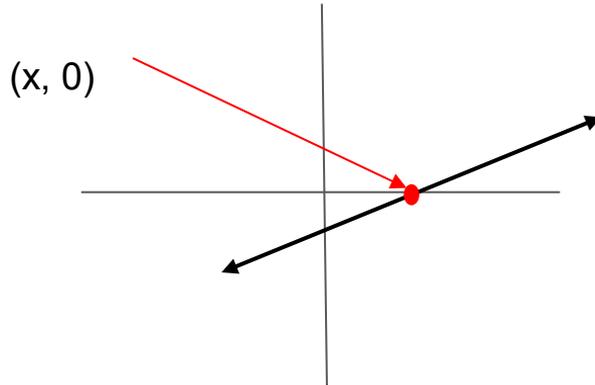
Point**Zero of a function**

x-value when $f(x) = 0$

Example:

$$g(x) = x - 2$$

$g(2) = 2 - 2 = 0$, so 2 is a zero of the function

x-intercept**Zero**

0

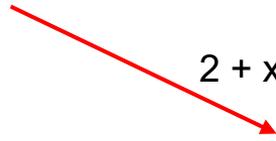
Equal

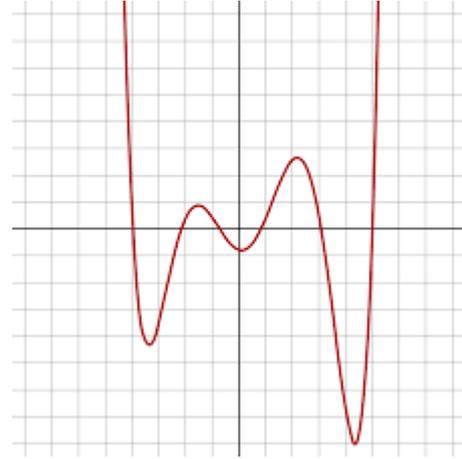
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Examples: $5 = 2 + 3$

$$3x + 6 = 3(x + 2)$$

Solution(s)

$$2 + x = 5$$

$$x = 3$$

Graph**Equation**

Examples:

$$f(x) = x^2 + 7x$$

$$2x - 5 = 0$$

$$10 = 7x + 5$$

$$y = 3x - 7$$