ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 3, Exponential and Logarithmic Functions

CONNECTIONS: Michigan Academic Standards for Mathematics

EXAMPLE CONTEXT FOR LANGUAGE USAGE: In pairs, both students will be given the same collection of logarithmic and exponential graphs. Partner 1 secretly chooses a graph and listens to Partner 2 ask yes/no questions in order to narrow down the graphs and eventually identify the chosen graph. This game is similar to 20 questions or the game "Guess Who". (If you are not familiar with this type of game, visit: t.) There are many variations to how this activity could be structured as long as the students are asking yes/no questions which incorporate the mathematical vocabulary of the unit. Example questions could be “Is the function increasing?”, “Does the function have a vertical asymptote?”, “Does it have an x-intercept at (5,0)?”.

There is a variety of mathematical reasoning and language that students can use to analyze and describe qualitative features of functions and their graphs. Some language has been suggested and scaffolded in the strand and associated supports. However, additional reasoning, such as transformations of functions, can also be used. In that case, an additional reference sheet to support the academic language related to transformations of functions could also be provided for students as necessary.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **SYNTHESIZE** questions about logarithmic and exponential functions in order to identify a graph by its qualitative features.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Synthesize questions about logarithmic and exponential functions asked multiple times while gesturing (e.g., moving hands in the direction of the curve), using an anchor chart, an illustrated reference sheet and clarifying language with a partner, after listening to students at higher levels of language proficiency model the process and the language.	Synthesize questions about logarithmic and exponential functions asked multiple times while gesturing (e.g., moving hands in the direction of the curve), using an anchor chart, an illustrated reference sheet and clarifying language with a partner, after listening to students at higher levels of language proficiency model the process and the language.	Synthesize questions asked multiple times about logarithmic and exponential functions using an anchor chart, an illustrated reference sheet, and clarifying language with a partner.	Synthesize questions asked about logarithmic and exponential functions using an anchor chart and clarifying language with a partner.	Synthesize questions asked about logarithmic and exponential functions using an anchor chart.	

ELD STANDARD 3: The Language of Mathematics			MAISA Algebra 2, Unit 3, Exponential and Logarithmic Functions			

EXAMPLE CONTEXT FOR LANGUAGE USAGE: This strand addresses standard HSF-BF.B.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. The task is part of a unit that was adapted from Key Curriculum's Discovering Advanced Algebra* book. The main purpose of the task is for students to discover that exponential and logarithmic functions are inverses of one another and what that relationship means graphically. A strong understanding of this relationship will support the algorithmic task of solving equations using these inverses. This task is only a small part of a bigger unit where students discover mathematical relationships through investigation, so this support shows a completed example of what a student-generated table/graph would look like from which this speaking domain can be emphasized. Parts 1c and 1d of the task could also be foundational speaking tasks that ultimately lead to 1h on which this strand is focused. Note: students at Levels 1, 2, and 3 are given a transformation anchor chart. However, it is advisable to review, with all students, the terminology associated with transformations as the content of this task requires students to recognize that one function is a reflection (over the line y=x) of the other.

*Murdock, J., Kamischke, E., & Kamischke, E. (2004). Discovering advanced algebra: An investigative approach. Emeryville, CA: Key Curriculum Press.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** the table and graph of an exponential and logarithmic function to **DESCRIBE** their relationship as inverses.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking/ Writing	Describe to a partner in short phrases or sentences the similarities and differences between the graphs of an exponential function and its logarithmic inverse (e.g., y=10^x and y=log x) using a function anchor chart, a transformation anchor chart, and sentence frames with choices while pointing to the anchor chart and/or table or graph on the lesson sheet. The graph of ______ (equation) ______ (is/ is not) a function. [use to describe each function] The graph of _______(equation) ______ (has/ does not have) a ________(vertical/ horizontal) asymptote. [use to describe each function]	Describe to a partner in simple sentences the similarities and differences between the graphs of an exponential function and its logarithmic inverse (e.g., y=10^x and y=log x) using a function anchor chart, a transformation anchor chart, and sentence frames with choices while pointing to the anchor chart and/or table or graph on the lesson sheet. The graph of ______ (equation) ______ (is/ is not) a function. [use to describe each function] The graph of _______(equation) ______ (has/ does not have) a ________(vertical/ horizontal) asymptote. [use	Describe to a partner in multiple complete sentences the similarities and differences between the graphs of an exponential function and its logarithmic inverse (e.g., y=10^x and y=log x) using a function anchor chart, a transformation anchor chart, sentence stems/frames with choices and a suggested word list (e.g., function, asymptote, x-coordinate, y-coordinate, ordered pair, domain, range, horizontal/vertical, reflection/translation/rotation). [Open-ended stem] The graphs are ______ (similar/different) because..... [repeat as needed to describe both] [Sentence stems/frames with additional scaffolds.]	Describe to a partner in compound and/or complex sentences the similarities and differences between the graphs of an exponential function and its logarithmic inverse (e.g., y=10^x and y=log x) using a function anchor chart and suggested word list (e.g., function, asymptote, x-coordinate, y-coordinate, ordered pair, domain, range, horizontal/vertical, reflection/translation/rotation). E.g., "Each graph is a function and has an asymptote. The two graphs have the same basic curved shape. The order of the x-coordinate and y-coordinate is	Describe to a partner in compound and/or complex sentences the similarities and differences between the graphs of an exponential function and its logarithmic inverse (e.g., y=10^x and y=log x) using a function anchor chart and required word list (e.g., function, asymptote, coordinates/ordered pair, domain, range, reflection/translation/rotation). E.g., "Each graph is a function and has an asymptote. The two graphs have the same basic curved shape. The order of the x-coordinate and y-coordinate is switched in the ordered	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	The _____(x/y) values of this table [pointing to the corresponding table] are the_____(x/y) values of this table [pointing to the corresponding table]. The ________ (domain/ range) of _______ (equation) is the same as the ________ (domain/ range) of _______ (equation). One graph is a __________(translation/reflection/rotation) of the other.	to describe each function] When you ___________(keep/switch) the numbers in the ordered pairs of one graph, you get the other. The ________ (domain/ range) of _______ (equation) is the same as the ________ (domain/ range) of _______ (equation). One graph is a __________(translation/reflection/rotation) of the other.	The graph of ______ (equation) ______ (is/ is not) a function. [use to describe each function] The graph of _______ (equation) ______ (has/ does not have) a ________(vertical/ horizontal) asymptote. [use to describe each function] When the numbers in the ordered pairs of one graph are switched.... The ________ (domain/ range) of _______ (equation) is the same as the ________ (domain/ range) of _______ (equation). The ___(vertical / horizontal) ____ asymptote of ___ (equation)___ is __(x = __ or y = __)__ and the ___(vertical / horizontal)____ asymptote of ___(equation)___ is __(x = __ or y = __)__. The first asymptote is a ___(vertical shift / horizontal shift / reflection)__ of the second. One graph is a __________ (translation/reflection/rotation) of the other.	switched in the ordered pairs of the graph of y equals log of x as compared to the ordered pairs of the graph of y equals 10 to the x power. This means the domain and range will be switched for the two functions. The graphs appear to be reflections over the line y equals x."	pairs of the graph of y equals log of x as compared to the ordered pairs of the graph of y equals 10 to the x power. This means the domain and range will be switched for the two functions. The graphs appear to be reflections over the line y equals x."	

ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 3, Exponential and Logarithmic Functions

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will read and analyze a passage that introduces a real-world context for logarithms. Two of the most commonly used examples are the decibel scale for sound intensity and the Richter scale for earthquake intensities. Students will understand the usefulness of using a logarithmic scale when intensities can vary from extremely small to extremely large values. This understanding of real-world applications will help students make more sense of the procedural skills used to analyze logarithmic functions and equations. The sample text is taken from the CPM Core Connections Algebra 2 textbook: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). Core Connections: Algebra 2. Sacramento, CA: CPM Educational Program.

Note that in the strand written below, the language function is to analyze a linguistically complex piece of mathematical text, but the more complex words in the text would be glossed for students at Levels 3 and 4, and the text would be simplified for students at Levels 1 and 2. For similar reading tasks, a teacher might re-write the text excerpt in simplified form, or alternatively may guide the student in highlighting just the key words in the original excerpt in order to simplify it.

Students may need explicit instruction on how to work with a partner during a reading activity. Since the academic language of the example reading task is highly specialized and mathematically sophisticated, all students would benefit from additional tools to aid in comprehension. One strategy that helps students process dense academic text is "Talking to the Text". This strategy will help students talk to one another about the text as well as aid in individual comprehension. This link provides an overview of the "Talking to the Text" strategy: https://rtc.instructure.com/courses/1056743/pages/talking-to-the-text , https://www.teachbetweenthelines.org/read-from-the-blog-1/2018/8/2/z22gp5d98nk3dkekifu1vqr2hf516z

The corresponding supports for this strand have been developed for the introduction of the problem/task. Additional scaffolds and supports will need to be developed for each of the problems within the task.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** a linguistically complex piece of mathematical text in order to **APPLY** this knowledge to complete an inquiry-based lesson.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Analyze a simplified mathematical text in order to understand and apply the information to complete an inquiry-based lesson while using an illustrated word list and working with a small group. Sample simplified text may be found in the supports for this unit.	Analyze a simplified mathematical text in order to understand and apply the information to complete an inquiry-based lesson while using an illustrated word list and working with a small group. Sample simplified text may be found in the supports for this unit.	Analyze a glossed version of a linguistically complex mathematical text in order to understand and apply the information to complete an inquiry-based lesson, using an illustrated word list and working with a partner. Sample glossed version of text may be found in the supports for this unit.	Analyze a glossed version of a linguistically complex mathematical text in order to understand and apply the information to complete an inquiry-based lesson, working with a partner. Sample glossed version of text may be found in the supports for this unit.	Analyze a linguistically complex mathematical text in order to understand and apply the information to complete an inquiry-based lesson, working with a partner. Sample reading passage may be found in the supports for this unit.	

ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 3, Exponential and Logarithmic Functions

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students will explain how to use a table or a graph to solve an equation in the form of f(x) = g(x), where at least one of the functions is logarithmic or exponential. This strand addresses the content standard HSA-REI.D.11: "Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately..." While all students should be encouraged to reflect on their work as a scaffold for writing and sometimes show work, students at levels 1 and 2 rely on their student-produced labeled work to write about a method to solve an equation. This sample task could be used for speaking or writing as the scaffolds would be the same or similar.

Students at all levels of proficiency may benefit from the opportunity to talk through their thinking with a partner. This allows students oral practice with the language of mathematics as related to logarithmic or exponential functions before producing the written product. Additionally, for more complex systems, students should use technology to produce graphs of the system. The student-generated sketch documents what was produced by technology and provides both a basis for mathematical reasoning and a language scaffold for students.

The writing tasks are intended to build connections between graphs, tables, and algebraic representations. The illustrated word bank assists students with word meanings and the task helps to create deeper understanding of words like solution. As such, the familiar representation of solution is given on the illustrated word bank. As the students complete the task, discuss as a class the connections between algebraic, graphical, and tabular solutions.

COGNITIVE FUNCTION: Students at all levels of English language proficiency analyze an equation containing two functions and **EXPLAIN** a method for solving using a table or a graph.

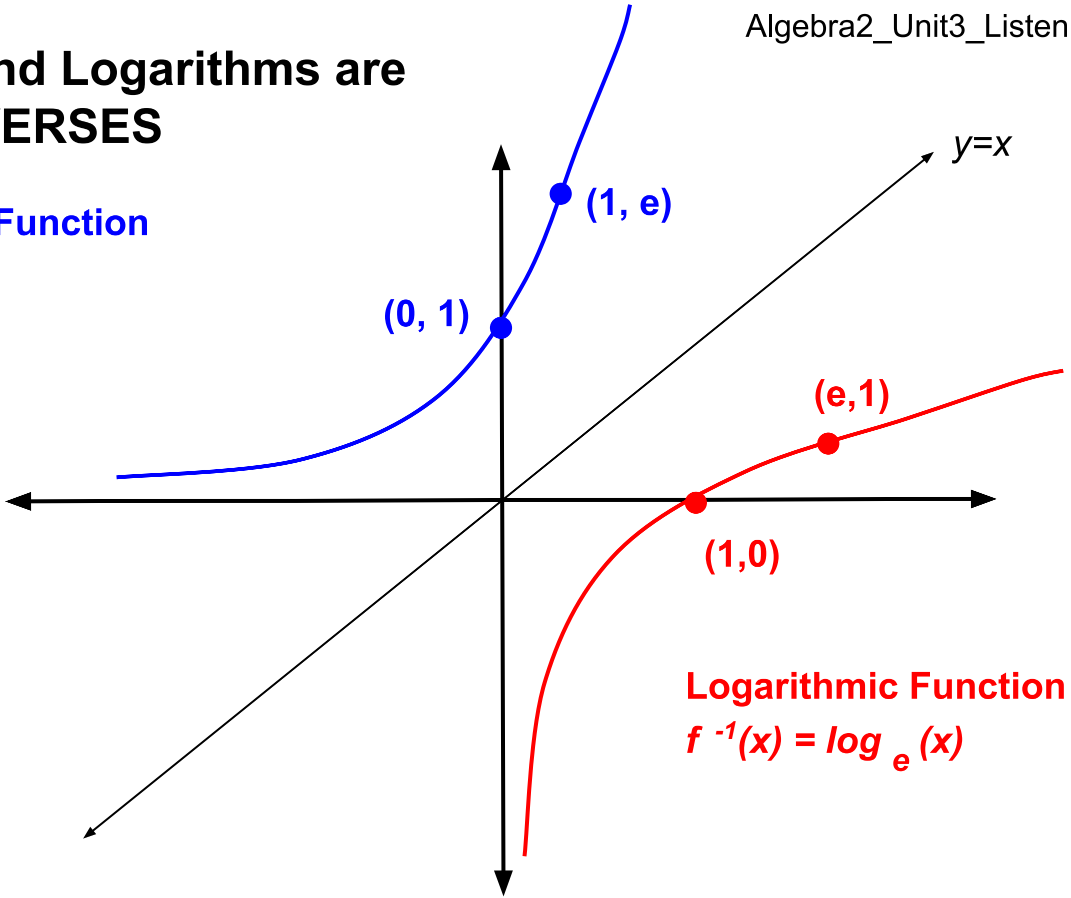
	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Writing	Explain in words and/or phrases, accompanied by a student-produced labeled table or sketch, a method for using a table or a graph to solve an equation in the form f(x) = g(x) or f(x)-g(x)=0 in which at least one of the functions is exponential or logarithmic, using sentence frames with choices and choosing appropriate terms on a function families anchor chart and an illustrated word bank. This is a(n) _______ function and a(n)_________ function. (choose from the anchor chart)	Explain in simple sentences, accompanied by a student-produced labeled table or sketch, a method for using a table or a graph to solve an equation in the form f(x) = g(x) or f(x)-g(x)=0 in which at least one of the functions is exponential or logarithmic, using sentence frames with choices and choosing appropriate terms on a function families anchor chart and an illustrated word bank. This is a(n) _______ function and a(n)_________ function. (choose from the anchor chart)	Explain in complete sentences, accompanied by a student-produced table or sketch, a method for using a table or a graph to solve an equation in the form f(x) = g(x) or f(x)-g(x)=0 in which at least one of the functions is exponential or logarithmic, referring to a function families anchor chart and using sentence frames/stems with a suggested word list (e.g., equation, function, equivalent equation, table, graph, intersect, coordinates, solutions, approximate). The equation contains a(n) ______function and a(n) ______function. (choose from	Explain in compound and/or complex sentences, accompanied by a student-produced table or sketch, a method for using a table or a graph to solve an equation in the form f(x) = g(x) or f(x)-g(x)=0 in which at least one of the functions is exponential or logarithmic, referring to a function families anchor chart and using a suggested word list (e.g., equation, function, rearrange, table, graph, intersect, coordinates, solutions, approximate).	Explain in compound and/or complex sentences, accompanied by a student-produced table or sketch, a method for using a table or a graph to solve an equation in the form f(x) = g(x) or f(x)-g(x)=0 in which at least one of the functions is exponential or logarithmic, referring to a function families anchor chart. E.g., [Given: Explain how to use a table or graph to solve the equation log2x + 3x = 0 or log2x = -3x.]	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Continued	First _________ (write equivalent equation, graph, make a table). Next _________ (write equivalent equation, graph, make a table). I see the solution(s) on the ______ (table/graph). The solution(s) is/are _____. OR There is no solution. [Students should sketch the representation they are using to solve the equation and provide words or short phrases to label the solution or process.]	First _________ (write equivalent equation, graph, make a table). Next _________ (write equivalent equation, graph, make a table). I see the solution(s) on the ______ (table/graph). The solution(s) is/are _____. OR There is no solution. [Students should sketch the representation they are using to solve the equation and provide words or short phrases to label the solution or process.]	the anchor chart) First I would ... Then I would ... Next I would ... The solutions to the equation are ... (OR There is no solution.)	E.g., [Given: Explain how to use a table or graph to solve the equation log2x + 3x = 0 or log2x = -3x.] "Because the equation contains a logarithmic function and a linear function, I can write an equivalent equation so that the logarithmic function is on one side of the equation and the linear function is on the other side. If I create a table or a graph of each function, I can find the points they have in common, and the x-coordinates will be the solutions to the equation." [OR There is no solution.]	"Because the equation contains a logarithmic function and a linear function, I can write an equivalent equation so that the logarithmic function is on one side of the equation and the linear function is on the other side. If I create a table or a graph of each function, I can find the points they have in common, and the x-coordinates will be the solutions to the equation." [OR There is no solution.]	

Algebra2_Unit3_Listening_AnchorChart

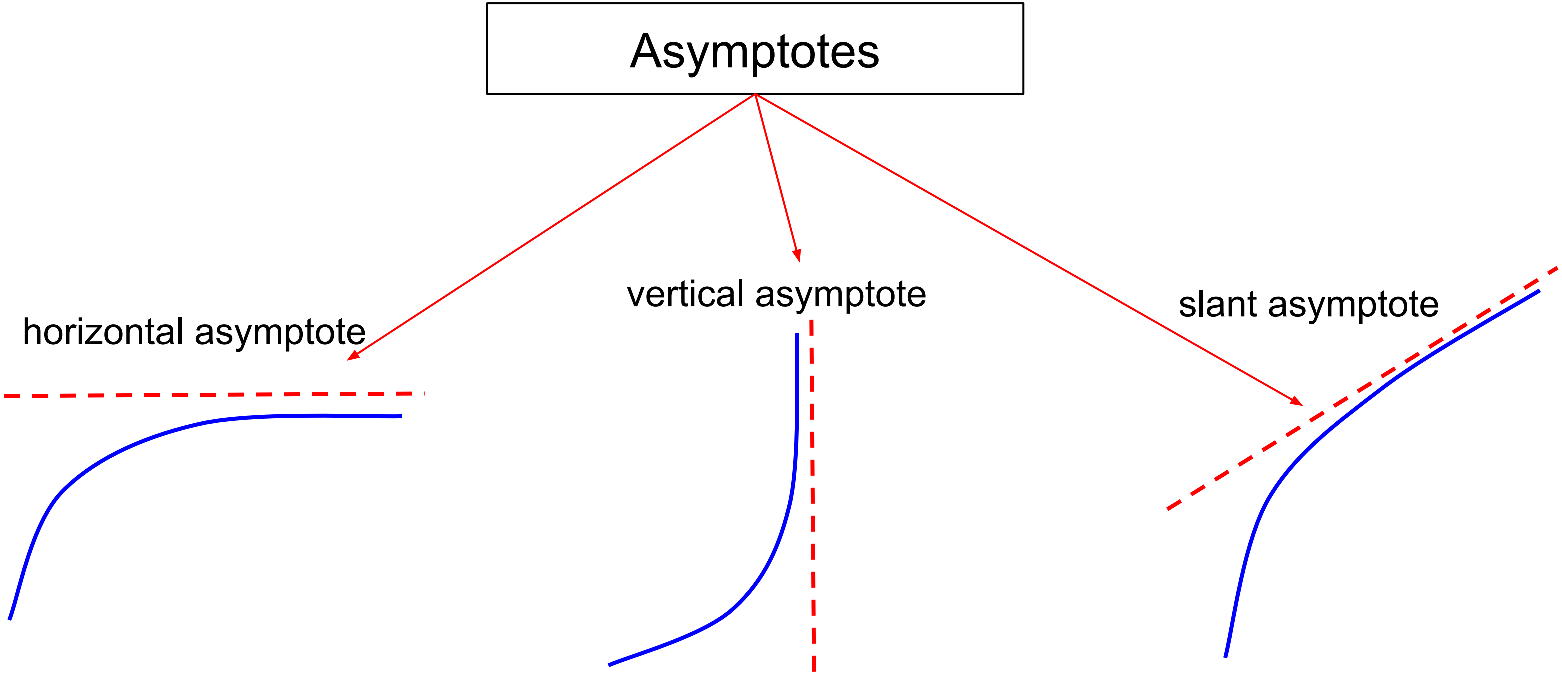
Exponential and Logarithms are INVERSES

Exponential Function
$f(x) = e^x$

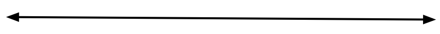


Logarithmic Function
$f^{-1}(x) = \log_e(x)$

Algebra2_Unit3_Listening_AnchorChart

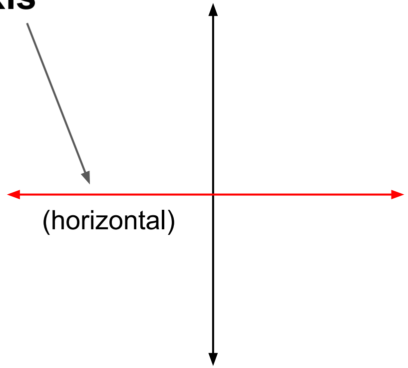


Horizontal

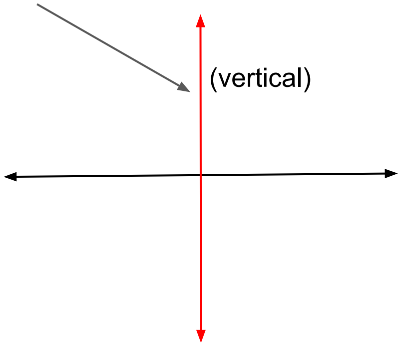


Vertical

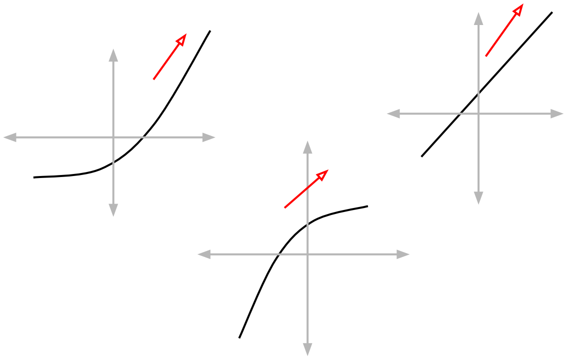
x-axis



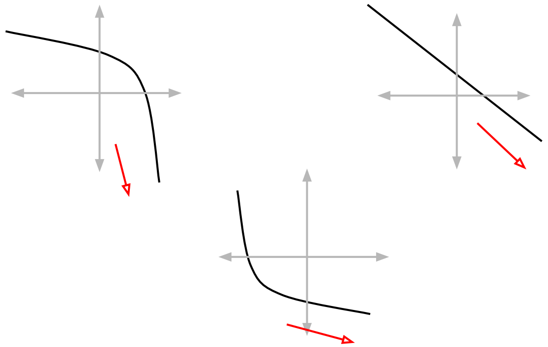
y-axis



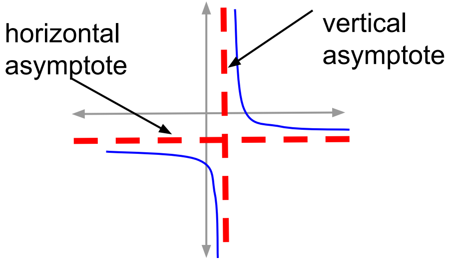
Increasing (read left to right)

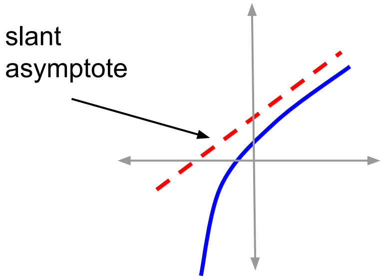


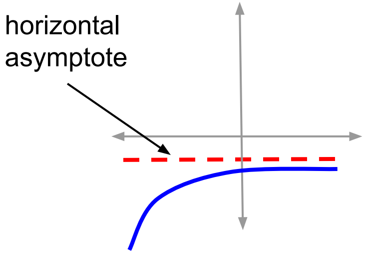
Decreasing (read left to right)

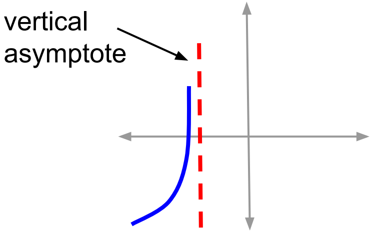


Asymptote - A line that a curve approaches, as it heads towards infinity” or “a horizontal, vertical, or slanted line that a graph approaches but never touches



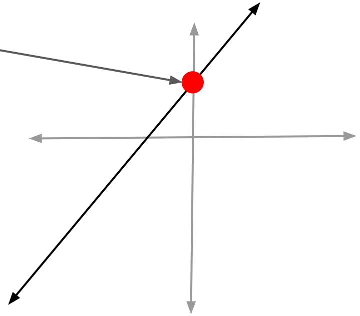






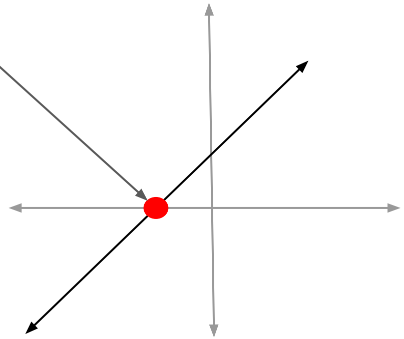
y-intercept

$(0, y)$



x-intercept

$(x, 0)$



Logarithm Investigation #2

1) Use your calculator table to investigate the following:
(Round answers to 2 decimal places when necessary.)

a) Complete the table for $y = 10^x$

x	$y = 10^x$

b) Use the *y*-values from your first table as *x*-values in the table below. Use these *x*-values and equation $y = \log x$ to compute *y*.

x	$y = \log x$

c) Consider the equation $y = 10^x$. [Use your calculator table to investigate x-values that are smaller and smaller.]

What happens to the y-value as the x-value decreases?

Does there appear to be a limit?

d) Consider the equation $y = \log x$. [Use your calculator table to investigate x-values that are smaller and smaller.]

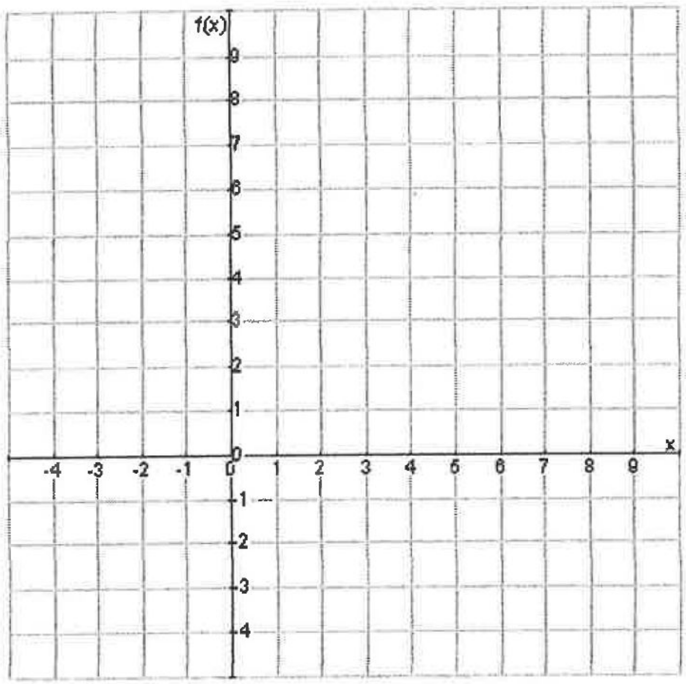
What happens to the *y*-value as the *x*-value gets closer and closer to 0, but stays positive?

Does there appear to be a limit?

e) What is the relationship between the limit in c) and the limit in d)?

Adapted from Key Curriculum's Discovering Advanced Algebra
Murcock, J., Kamischke, E., & Kamischke, E., (2004). Discovering advanced algebra: An investigative approach. Emeryville, CA: Key Curriculum Press.

f) Use the points from your tables above and your findings in c) and d) to sketch the graph of each equation.



g) Use your calculator with the window shown in the grid above to check your graphs.

h) Refer to the graphs to fill in the table below.

	$y = 10^x$	$y = \log x$
Is the graph a function?		
What is the domain?		
What is the range?		
Is the function increasing or decreasing?		

i) Compare the two graphs. What are the similarities? What are the differences?

Logarithm Investigation #2

1) Use your calculator table to investigate the following:
(Round answers to 2 decimal places when necessary.)

a) Complete the table for $y = 10^x$

x	$y = 10^x$
-1	.1
0	1
0.5	3.16
1	10

b) Use the y-values from your first table as x-values in the table below. Use these x-values and equation $y = \log x$ to compute y.

x	$y = \log x$
.1	-1
1	0
3.16	.5
10	1

c) Consider the equation $y = 10^x$. [Use your calculator table to investigate x-values that are smaller and smaller.]

What happens to the y-value as the x-value decreases?

Does there appear to be a limit?

As the x-value gets bigger in the negative direction, the y value gets smaller and smaller but remains positive. The limit appears to be 0.

d) Consider the equation $y = \log x$. [Use your calculator table to investigate x-values that are smaller and smaller.]

What happens to the y-value as the x-value gets closer and closer to 0, but stays positive?

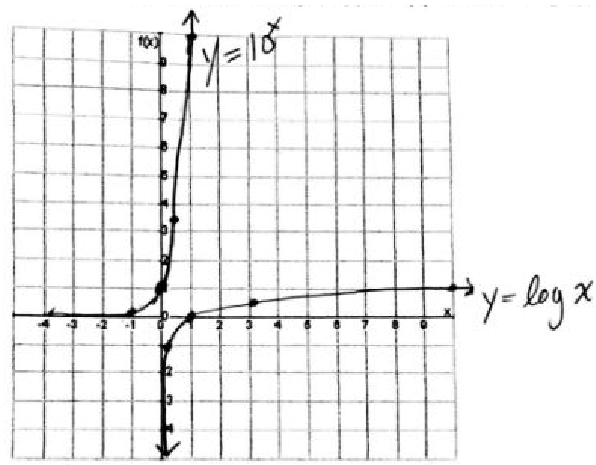
Does there appear to be a limit?

As the x-value gets closer to 0 from the positive direction, the y value gets bigger and bigger in the negative direction. There does not appear to be a limit.

Adapted from Key Curriculum's Discovering Advanced Algebra
Murcock, J., Kamischke, E., & Kamischke, E., (2004). Discovering advanced algebra: An investigative approach. Emeryville, CA: Key Curriculum Press.

e) What is the relationship between the limit in c) and the limit in d)?

f) Use the points from your tables above and your findings in c) and d) to sketch the graph of each equation.



g) Use your calculator with the window shown in the grid above to check your graphs.

h) Refer to the graphs to fill in the table below.

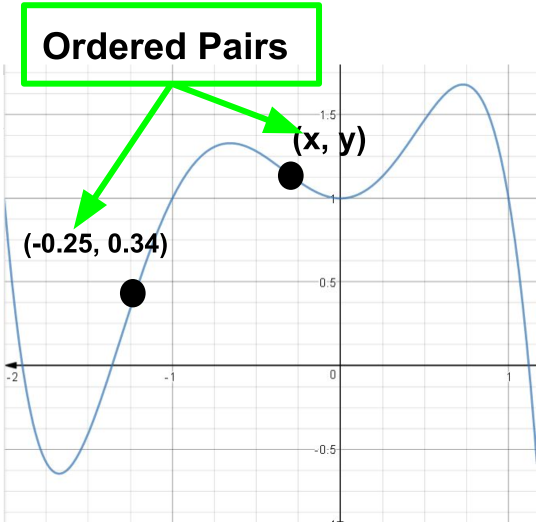
	y = 10^x	*y = log x*
Is the graph a function?	yes	yes
What is the domain?	$\{x \mid -\infty < x < \infty\}$ or All $\mathbb{R}$ #'s	$\{x \mid x > 0\}$
What is the range?	$\{y \mid y > 0\}$	$\{y \mid -\infty < y < \infty\}$ or All $\mathbb{R}$ #'s
Is the function increasing or decreasing?	**increasing**	**increasing**

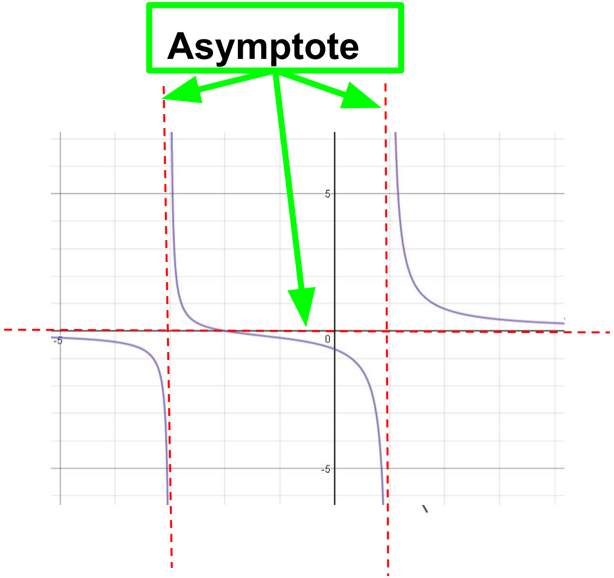
i) Compare the two graphs. What are the similarities? What are the differences?

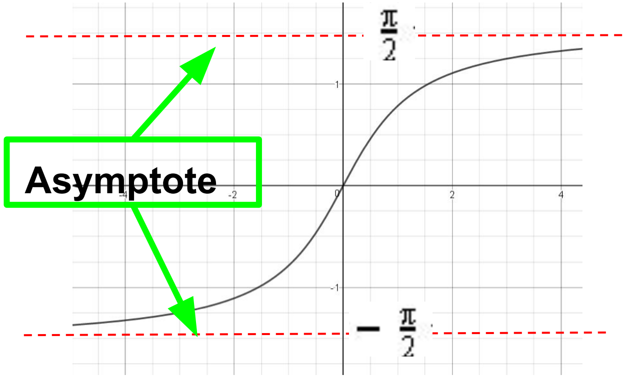
Each graph is a function and has an asymptote. The two graphs have the same basic shape (curve. The order of the x-coordinate and y-coordinate are switched in the ordered pairs of the graph of $y = \log x$ as compared to the ordered pairs of the graph of $y = 10^x$. This means the domain and range will be switched for the two functions. The graphs appear to be reflections over the line $y = x$.

FUNCTION EXAMPLES

Asymptote - a line that the graph approaches but never touches

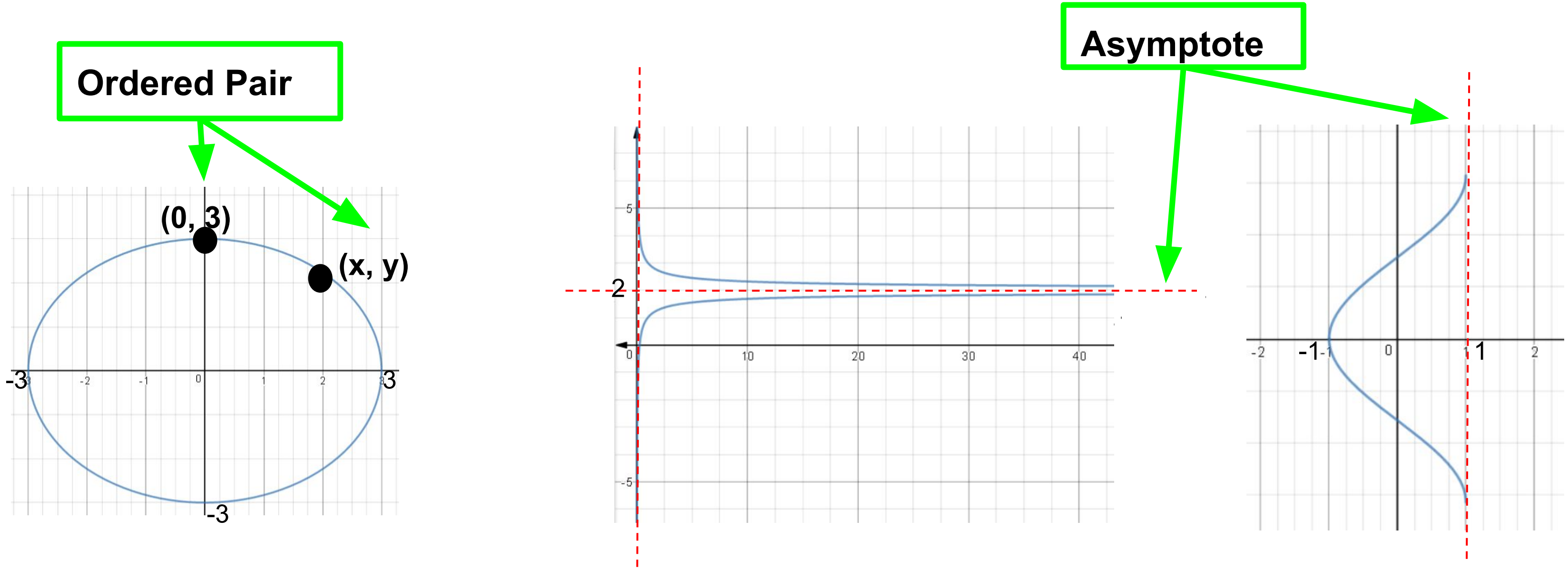




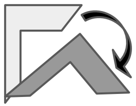
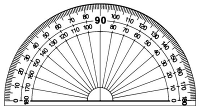
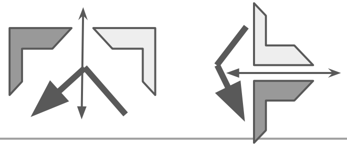
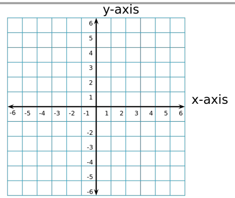
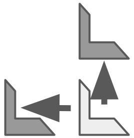
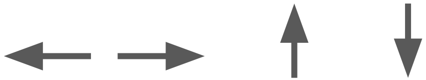
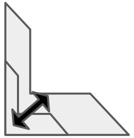
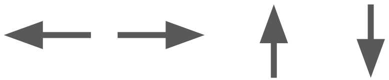


Domain:	{x|x = all real numbers}	{x| $-\infty < x < -3$ *and* $-3 < x < 1$ *and* $1 < x < \infty$}	{x|x = all real numbers}
Range:	{y|y = all real numbers}	{y|y = all real numbers}	{y| $-\frac{\pi}{2} < y < \frac{\pi}{2}$ }

NOT FUNCTION EXAMPLES



Domain:	$\{x \mid -3 \leq x \leq 3\}$	$\{x \mid 0 < x < \infty\}$	$\{x \mid -1 \leq x < 1\}$
Range:	$\{y \mid -3 \leq y \leq 3\}$	$\{y \mid -\infty < y < 2 \text{ and } 2 < y < \infty\}$	{y|y = all real numbers}

TRANSFORMATION	HOW	HOW MUCH
Rotation/Rotate 	clockwise / counterclockwise around the point <u>(x ,y)</u>  	______(#) degrees 
Reflection/Reflect 	over the ______(x-axis/ y-axis/ line____) 	
Translation/Translate 	shifts/moves (left / right // up / down) (horizontally // vertically) 	______(#) units
Stretch or Shrink 	horizontally / vertically 	by a factor of ______(#)

CPM Algebra 2 Textbook Page 377-378 (Original Version with Table):

Have you ever had someone tell you that you were speaking too loudly (or too softly) or that the volume on a television was turned up too high (or down too low)? While sensitivity to noise can vary from person to person, in general, people can hear sounds over an incredible range of loudness.

Sound intensity is measured in physical units of watts per square centimeter. But the loudness is typically reported in units of relative intensity called decibels. The next table shows intensity values for a variety of familiar sounds and the related number of decibels - as measured at common distances from the sources.

Study the sound intensity values, sources, and decibel ratings given in the table.

a) Are the intensity and decibel numbers in the order of loudness that you would expect for the different familiar sources?
b) What pattern do you see relating the sound intensity values (watts/cm^2) and the decibel numbers?

Sound Intensity (in watts/cm^2)	**Noise Source**	**Relative Intensity** (in decibels)
10^{3}	Military Rifle	150
10^{2}	Jet plane (30 meters away)	140
10^{1}	[Level at which sound is painful]	130
10^{0}	Amplified rock music	120
10^{-1}	Power tools	110
10^{-2}	Noisy kitchen	100
10^{-3}	Heavy traffic	90
10^{-4}	Traffic noise in a small car	80
10^{-5}	Vacuum cleaner	70
10^{-6}	Normal conversation	60
10^{-7}	Average home	50
10^{-8}	Quiet conversation	40
10^{-9}	Soft whisper	30
10^{-10}	Quiet living room	20
10^{-11}	Quiet recording studio	10
10^{-12}	[Barely Audible]	0

In this lesson, you will learn about logarithms - the mathematical idea used to express sound intensity and to solve a variety of problems related to exponential functions and equations.

Investigation 1: How Loud is Too Loud?

Your analysis of the sound intensity data might have suggested several different algorithms for converting watts per square centimeter into decibels. For example, if the intensity of a sound is 10^x watts/cm^2, its loudness in decibels is 10x + 120.

The key to discovery of this conversion rule is the fact that all sound intensities were written as powers of 10. What would you have done if the sound intensities had been written as numbers like 3.45 watts/cm^2 or 0.0023 watts/cm^2?

As you work on problems of this investigation, look for an answer to this question: How can any positive number be expressed as a power of 10?

Source: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). Core Connections: Algebra 2. Sacramento, CA: CPM Educational Program.

Source: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). Core Connections: Algebra 2. Sacramento, CA: CPM Educational Program.

Note: Words with numerical superscripts are found in the illustrated word list.

CPM Algebra 2 Textbook Page 377-378 (Glossed Version with Table):

Have you ever had someone tell you that you were speaking too loudly[4] (or too softly[3]) or that the volume[2] on a television was turned up too high (or down too low)? While *sensitivity to noise*[9] [ability to hear] can *vary*[4.5] [be different] from person to person, in general, people can hear sounds over an *incredible* [very large] *range*[1] *of loudness* [from very quiet to very loud].

Sound *intensity*[5] [strength] is measured in physical units[6] of watts per square centimeter[7]. But the loudness is typically *reported* [usually told or written] by relative intensity[5] in units[6] called decibels. The next table[8] shows intensity[5] *values* [numbers] for a *variety* [many different] of *familiar* [common] sounds and the related number of decibels - as measured at common distances from the *sources*[10] [things that make the sounds].

Sound Intensity (in watts/cm^2)	Noise Source	Relative Intensity (in decibels)
10^{3}	Military Rifle	150
10^{2}	Jet plane (30 meters away)	140
10^{1}	[Level at which sound is painful]	130
10^{0}	Amplified rock music	120
10^{-1}	Power tools	110
10^{-2}	Noisy kitchen	100
10^{-3}	Heavy traffic	90
10^{-4}	Traffic noise in a small car	80
10^{-5}	Vacuum cleaner	70
10^{-6}	Normal conversation	60
10^{-7}	Average home	50
10^{-8}	Quiet conversation	40
10^{-9}	Soft whisper	30
10^{-10}	Quiet living room	20
10^{-11}	Quiet recording studio	10
10^{-12}	[Barely Audible]	0

Study the sound intensity[5] *values* [numbers], the sources[10], and the *decibel ratings* [numbers] given in the table[8].

a) Are the intensity[5] and decibel numbers in the order of loudness that you would expect for the different familiar sources[10]?

b) What *pattern*[11] [relationship[12]] do you see *relating* [connecting] the sound intensity[5] values (watts/cm^2) and the decibel numbers?

In this lesson, you will learn about logarithms - the mathematical idea used to *express* [describe] sound intensity[5] and to solve a *variety of* [many different] problems *related* [connected] to exponential functions[13] and equations[14].

Investigation 1: How Loud is Too Loud?

Your *analysis*[15] [study, looking carefully] of the sound intensity[5] *data* [numbers in the table[8]] might have *suggested* [given you an idea of] several different *algorithms*[16] [ways, methods] for *converting*[17] [changing] watts per square centimeter[7] into decibels. For example, if the intensity[5] of a sound is 10^x watts/cm^2, its loudness in decibels is 10x + 120.

The *key* [most important thing to help you solve a problem] to *discovery* [finding] of this *conversion rule* [method or equation[14]] is the fact that all sound intensities were written as powers of 10. What would you have done if the sound intensities had been written as numbers like 3.45 watts/cm^2 or 0.0023 watts/cm^2?

As you work on problems of this *investigation* [lesson], look for an answer to this question: How can any positive number be *expressed* [written] as a power of 10?

Source: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). *Core Connections: Algebra 2*. Sacramento, CA: CPM Educational Program.

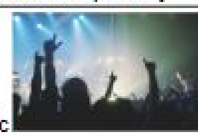
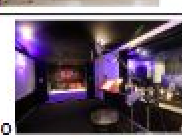
CPM Algebra 2 Textbook Page 377-378 (Simplified Version with Table):

Note: Words with numerical superscripts are found in the illustrated word list.

People can hear a wide range[1] of volumes[2] from very quiet[3] to very loud[4]. The intensity (strength)[5] of sound is measured physically in units[6] of watts per square centimeter[7]. But loudness is usually described in units called decibels.

The table[8] shows the intensity values for many common sounds and the loudness in decibels, measured at a normal distance.

Look carefully[15] at the sound intensity numbers, the sources, and the decibels given in the table.

Sound Intensity (in watts/cm^2)	Noise Source	Relative Intensity (in decibels)
10^{3}	Military Rifle 	150
10^{2}	Jet plane (30 meters away) 	140
10^{1}	[Level at which sound is painful]	130
10^{0}	Amplified rock music 	120
10^{-1}	Power tools 	110
10^{-2}	Noisy kitchen 	100
10^{-3}	Heavy traffic 	90
10^{-4}	Traffic noise in a small car	80
10^{-5}	Vacuum cleaner 	70
10^{-6}	Normal conversation	60
10^{-7}	Average home	50
10^{-8}	Quiet conversation	40
10^{-9}	Soft whisper	30
10^{-10}	Quiet living room 	20
10^{-11}	Quiet recording studio 	10
10^{-12}	[Barely Audible]	0

a) Does the order in the noise[9] source[10] list make sense?

b) What pattern[11] or relationship[12] can you see connecting the intensity numbers and the decibel numbers?

Source: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). *Core Connections: Algebra 2*. Sacramento, CA: CPM Educational Program.

In this lesson you will learn about logarithms connected to exponential functions[13] and equations[14].

Investigation 1: How Loud is Too Loud?

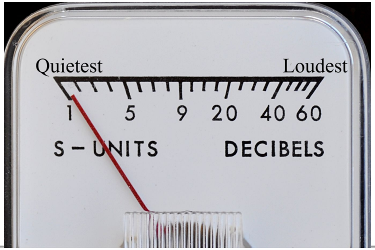
When you looked[15] at the numbers in the table, did you see a way[16] to change[17] the intensity values (watts/cm^2) to decibels? For example, if the intensity[5] of a sound is 10^x watts/cm^2, its loudness[4] in decibels is 10x + 120.

One thing that made this rule[16] easier to find was that all the intensity numbers were written as powers of 10. What would you do if the sound intensities were written as numbers like 3.45 watts/cm^2 or 0.0023 watts/cm^2?

As you work on these problems, look for an answer to this question: How can any positive number be written as a power of 10?

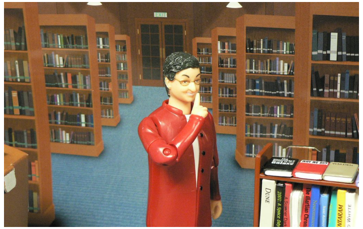
Source: Kysh, J., Baldinger, E., & Kassarjian, M. (2013). *Core Connections: Algebra 2*. Sacramento, CA: CPM Educational Program.

Range[1]



Volume[2]

Soft/ Quiet[3]

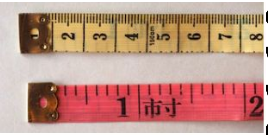


Loud[4]

Vary[4.5]

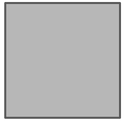
Intensity/ Strength[5]

Units[6]



centimeters (cm)
cm squared (cm^2)
watts (W)
watts/cm^2
decibels (dB)

Square Centimeter[7] (cm^2)



1 cm

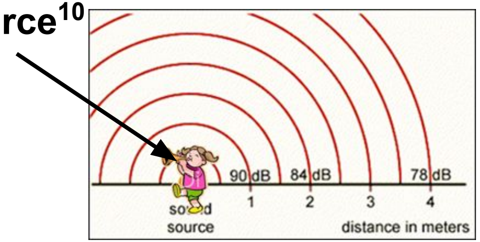
1 cm

Table[8]

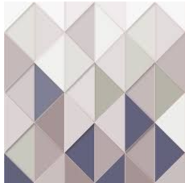
Square side length	Area of a square
3 cm	9 cm^2
4 cm	16 cm^2

Noise[9]

Source[10]

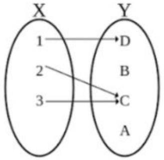


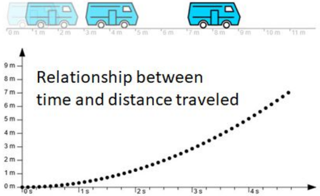
Pattern[11]



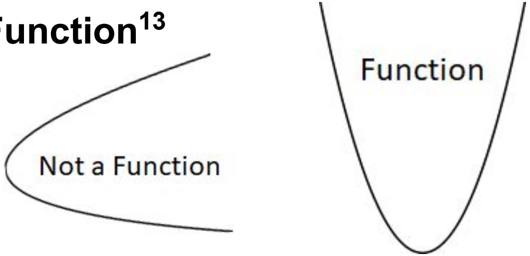
x	f(x)	
1	9	)-4
2	5	)-4
3	1	)-4
4	-3	)-4
5	-7	

Relationship[12]





Function[13]



Equation[14]

Y = 3x+4

A=

$$4^x = \left(\frac{3}{2}\right)^8$$

Analysis (study/ look carefully)[15]

Algorithm (process/way/rule)[16]

Converting (changing)[17]

1 kg ⟷ 1,000 g

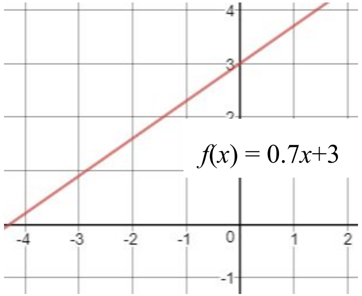
60 min ⟷ 1 hour

Rifle 	**Jet** 	**Rock Music** 
Power Tools  	**Kitchen** 	**Traffic** 
Vacuum 	**Living Room** 	**Recording Studio** 

Function Families $a \neq 0$

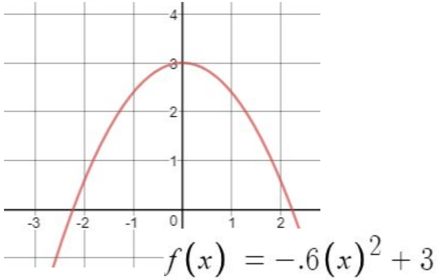
Linear Function

$f(x) = a(x-h)+k$



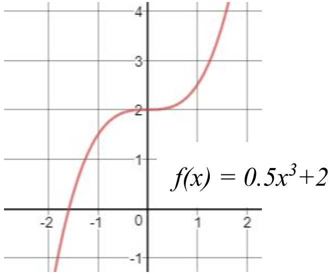
Quadratic Function

$f(x) = a(x-h)^2 + k$



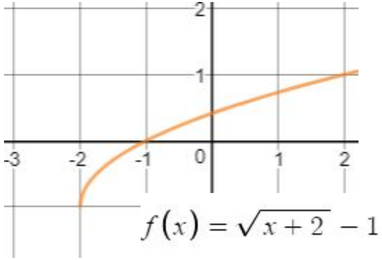
Cubic Function

$f(x) = a(x-h)^3 + k$



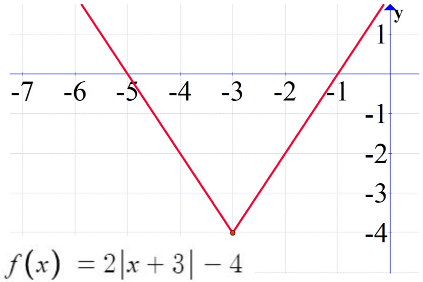
Square Root Function

$f(x) = a\sqrt{x-h} + k$



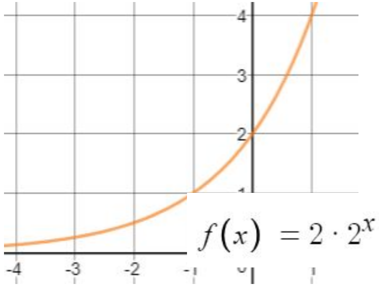
Absolute Value Function

$f(x) = a|x-h| + k$



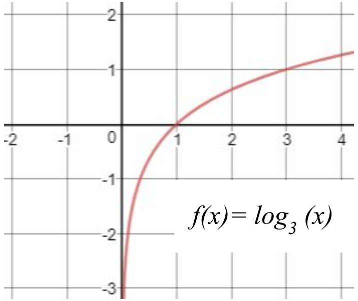
Exponential Function

$f(x) = a(b)^x$



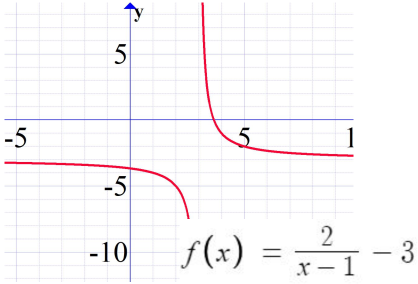
Logarithmic Function

$f(x) = a\log_b(x)$



Rational Function

$f(x) = \frac{a}{x-h} + k$



Function Families (Student-Generated Examples)

Linear Function	Quadratic Function	Cubic Function	Square Root Function
Absolute Value Function	Exponential Function	Logarithmic Function	Rational Function

Equation

$$3x + 2y = 18$$

$$log_2(6x) + 25x = 200$$

$$x^3 = \frac{x}{8}$$

Function

$$f(x) = ax^2 + b$$

$$f(x) = a(b)^x$$

Write Equivalent Equations

$$2\log x = 5$$ (see "Solving Equations")

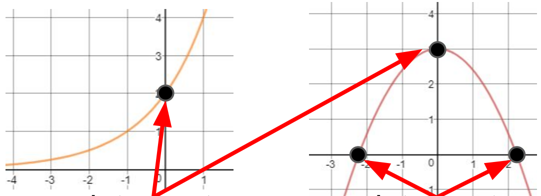
$$\log x = \frac{5}{2}$$

$$x = 10^{\frac{5}{2}}$$

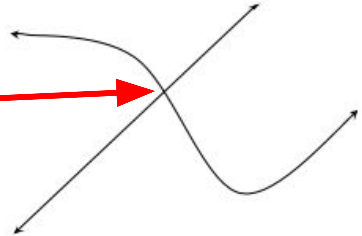
Table

x	y
-2	-4
0	0
2	4
4	8

Graph



Intersect



Coordinates

(-3, 9)

x-coordinate

y-coordinate

Solution

$$\begin{aligned} 21 + 3x &= 27 \\ -21 \quad & \quad -21 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \\ 21 + 3(2) &= 27 \checkmark \end{aligned}$$

Approximate

25/7 is 3.57142857…, which is about 3.6

Solving Equations

Write Equivalent Equations

$$2\log x = 5$$

$$\log x = \frac{5}{2}$$

$$x = 10^{\frac{5}{2}}$$

<u>Equivalent Equations Share the Same Solutions</u>

$$\text{Solution is } x = 10^{\frac{5}{2}}$$

Check that each equation has the same solution:

$$2\log 10^{\frac{5}{2}} = 5 \;\triangleright\; 2\left(\frac{5}{2}\right) = 5 \;\triangleright\; 5 = 5$$

$$\log 10^{\frac{5}{2}} = \frac{5}{2} \;\triangleright\; \frac{5}{2} = \frac{5}{2}$$

$$10^{\frac{5}{2}} = 10^{\frac{5}{2}}$$