Rational Functions Activity #1 Exploring Vertical Asymptotes and Roots with the Graphing Calculator

Exploration #1

a) Use your graphing calculator to graph $f(x) = x^2 + x - 6$ and g(x) = x + 2. As accurately as possible, graph these functions on the given grids below.



b) What are the roots of each function? Describe how you found them both graphically and algebraically.

c) Use your graphing calculator to graph the function $h(x) = \frac{f(x)}{g(x)}$. As accurately as possible, graph this function on the given grid below. Examine the graph of h(x) at the roots of f(x) (e.g. where f(x) = 0). What do you notice? Examine the graph of h(x) at the roots of g(x) (e.g. where g(x) = 0). What do you notice?



d) Graph the function $p(x) = \frac{g(x)}{f(x)}$ on the given grid. Examine the graph of p(x) at the roots of f(x). What do you notice? Examine the graph of p(x) at the roots of g(x). What do you notice?



Exploration #2

a) Use your graphing calculator to graph $f(x) = x^2 - x - 2$ and $g(x) = x^2 + 2x - 3$. As accurately as possible, graph these functions on the given grids below.



b) What are the roots of each function? Describe how you found them both graphically and algebraically.

c) Use your graphing calculator to graph the function $h(x) = \frac{f(x)}{g(x)}$. As accurately as possible, graph this function on the given grid below. Examine the graph of h(x) at the roots of f(x) (e.g. where f(x) = 0). What do you notice? Examine the graph of h(x) at the roots of g(x) (e.g. where g(x) = 0). What do you notice?



d) Graph the function $p(x) = \frac{g(x)}{f(x)}$ on the given grid. Examine the graph of p(x) at the roots of f(x). What do you notice? Examine the graph of p(x) at the roots of g(x). What do you notice?



Exploration #3

a) Use your graphing calculator to graph f(x) = (x-3)(x+2)(x-1) and g(x) = x-2. As accurately as possible, graph these functions on the given grids below.



b) What are the roots of each function? Describe how you found them both graphically and algebraically.

c) Use your graphing calculator to graph the function $h(x) = \frac{f(x)}{g(x)}$. As accurately as possible, graph this function on the given grid below. Examine the graph of h(x) at the roots of f(x) (e.g. where f(x) = 0). What do you notice? Examine the graph of h(x) at the roots of g(x) (e.g. where g(x) = 0). What do you notice?



d) Graph the function $p(x) = \frac{g(x)}{f(x)}$ on the given grid. Examine the graph of h(x) at the roots of f(x). What do you notice? Examine the graph of p(x) at the roots of g(x). What do you notice?



Summary and Conclusions

1. Given functions f(x) = (x-a)(x+b)(x-c) and g(x) = (x-d)(x+e) where a, b, c, d, and e are positive real numbers such that $a \neq b \neq c \neq d \neq e$,

a) Where are the roots of the function $h(x) = \frac{g(x)}{f(x)}$?

b) Where are the vertical asymptotes of the function $h(x) = \frac{g(x)}{f(x)}$?

c) Where are the roots of the function $p(x) = \frac{f(x)}{g(x)}$?

d) Where are the vertical asymptotes of the function $p(x) = \frac{f(x)}{g(x)}$?

- 2. What type of functions were f(x) and g(x)?
- 3. In all the explorations above, h(x) and p(x) are called *rational functions*. Can you formulate a definition for a rational function?
- 4. For any simplified rational function, what information can you obtain from the numerator?
- 5. For any simplified rational function, what information can you obtain from the denominator?