| ELD STANDARD 3: The Language of Mathematics | | | | | | MAISA Algebra 2, Unit 4, Rational Expressions & Functions |
|---|---|---|---|---|---|---|
| **CONNECTIONS**: Michigan Academic State Standards for Mathematics | | | | | | |
| **EXAMPLE CONTEXT FOR LANGUAGE USAGE:** Students will listen to descriptions of rational functions (asymptotes, x-intercepts, transformations) and write corresponding equations. There are many different configurations that teachers may choose for this listenting task. The following task could be structured as teacher to students or student to students, and in whole group, small group, or paired settings.<br><br>Using graphing technology allows students to make conjectures about the equations and efficiently check their thinking. Writing functions for a variety of descriptions assists students in making connections between the graphical and algebraic representations and also builds conceptual understanding about the effects of zeros in the numerator and denominator of rational functions.<br><br>**COGNITIVE FUNCTION:** Students at all levels of English language proficiency **SYNTHESIZE** a description of a rational function in order to write the equation. | | | | | | |
| | **Level 1 Entering** | **Level 2 Emerging** | **Level 3 Developing** | **Level 4 Expanding** | **Level 5 Bridging** | **Level 6 Reaching** |
| **Listening** | Synthesize a description of a rational function read aloud multiple times, with purposeful pauses and pointing to appropriate supports, to write the equation using an illustrated reference sheet and anchor chart, while working with a partner and checking work using technology.<br><br>E.g., "Write a rational function [pause and point] that has vertical asymptotes [pause and point] at $x = -3$ [pause] and $x = 2$ [pause] with an x-intercept [pause and point] at (-5, 0)." | Synthesize a description of a rational function read aloud multiple times, with purposeful pauses and pointing to appropriate supports, to write the equation using an illustrated reference sheet and anchor chart, while working with a partner and checking work using technology.<br><br>E.g., "Write a rational function [pause and point] that has vertical asymptotes [pause and point] at $x = -3$ [pause] and $x = 2$ [pause] with an x-intercept [pause and point] at (-5, 0)." | Synthesize a description of a rational function read aloud multiple times, with purposeful pauses, to write the equation using an illustrated reference sheet and anchor chart, while working with a partner and checking work using technology.<br><br>E.g., "Write a rational function [pause] that has vertical asymptotes [pause] at x = -3 [pause] and x = 2 [pause] with an x-intercept [pause] at (-5, 0)." | Synthesize a description of a rational function read aloud, with purposeful pauses, to write the equation using an anchor chart, working with a partner, and checking work using technology.<br><br>E.g., "Write a rational function [pause] that has vertical asymptotes [pause] at $x = -3$ [pause] and $x = 2$ [pause] with an x-intercept [pause] at (-5, 0)." | Synthesize a description of a rational function read aloud, with purposeful pauses, to write the equation using an anchor chart, working with a partner, and checking work using technology.<br><br>E.g., "Write a rational function [pause] that has vertical asymptotes [pause] at $x = -3$ [pause] and $x = 2$ [pause] with an x-intercept [pause] at (-5, 0)." | |

| **ELD STANDARD 3: The Language of Mathematics** | **MAISA Algebra 2, Unit 4, Rational Expressions & Functions** |
|---|---|

**EXAMPLE CONTEXT FOR LANGUAGE USAGE:** Students will compare the graphs of rational functions to determine how the parent graph of a function, *f(x)* , is transformed when different parameters of the function are changed (i.e. *f(x) + k* , *k f(x)* , *f(kx)* , and *f(x + k))* . This process could be repeated multiple times using both positive and negative *k* values. The teacher could decide whether to use this for speaking or writing as the scaffolds would/could be the same. The use of technology is encouraged and a sample activity sheet is provided.

The purpose for speaking (or writing) in this strand is to provide students opportunites to rehearse, or practice, their answers in pairs or small groups prior to speaking or presenting to the whole group or prior to writing formally.

Note that there is an added layer of linguistic complexity when students are speaking and writing about transformations. The same root word may be used for different purposes, with different endings, to represent the same idea or concept. For example, students are expected to understand when and how to use the root "translate" and to accurately use it when conveying the concept of "translation" as well as the action of 'translating". The teacher is encouraged to explicitly teach the different possible endings (e.g., -ing, -ed, -tion, etc.), as needed, for students at the lower levels of proficiency. Likewise, students should not be penalized if they use the correct root word with an incorrect ending throughout the formative cycle of the unit, or until they have received explicit instruction on this language use.

**COGNITIVE FUNCTION:** Students at all levels of English language proficiency **EXPLAIN** how changing a parameter of the function affects the graph.

| | **Level 1<br>Entering** | **Level 2<br>Emerging** | **Level 3<br>Developing** | **Level 4<br>Expanding** | **Level 5<br>Bridging** | **Level 6<br>Reaching** |
|---|---|---|---|---|---|---|
| **Speaking** | Explain in short phrases or sentences, while gesturing to communicate the transformation and pointing to the reference sheet, how changing a parameter of a rational function (i.e., *f(x) = 1/x* ) affects the graph using an anchor chart with additional glossing as needed, sentence frames with choices and working with a partner.<br><br>The transformation is a ____________(stretch/shrink/ translation/reflection).<br><br>**For Stretch/Shrink:**<br>The graph is _________(stretched/shrunk) __________(vertically/horizontally) by a factor of _____(#). | Explain in simple sentences, while gesturing to communicate the transformation and pointing to the reference sheet, how changing a parameter of a rational function (i.e., *f(x) = 1/x* ) affects the graph using an anchor chart with additional glossing as needed, sentence frames with choices and working with a partner.<br><br>The transformation is a ___________(stretch/shrink /translation/reflection).<br><br>**For Stretch/Shrink:**<br>The graph is _______(stretched/shrunk) __________(vertically/horizontally) by a factor of _____(#). | Explain in complete sentences how changing a parameter of a rational function (i.e., *f(x) = 1/x* ) affects the graph using an anchor chart, sentence stem, and working with a partner.<br><br>The transformation is a____(type of transformation)...(details of the transformation). | Explain in complete sentences how changing a parameter of a rational function (i.e., *f(x) = 1/x* ) affects the graph using an anchor chart and working with a partner.<br><br>E.g.,<br>a. "The graph of f of x equals 1 over x is translated to the left 3 units."<br><br>b. "The graph of f of x equals 1 over x is translated up 3 units."<br><br>c. "The graph of f of x equals 1 over x is stretched horizontally by a factor of 3." | Explain in complete sentences how changing a parameter of a rational function (i.e., *f(x) = 1/x* ) affects the graph using an anchor chart and working with a partner.<br><br>E.g.,<br>a. "The graph of f of x equals 1 over x is translated to the left 3 units."<br><br>b. "The graph of f of x equals 1 over x is translated up 3 units."<br><br>c. "The graph of f of x equals 1 over x is stretched horizontally by a factor of 3." | |

| | Level 1<br>Entering | Level 2<br>Emerging | Level 3<br>Developing | Level 4<br>Expanding | Level 5<br>Bridging | Level 6<br>Reaching |
|---|---|---|---|---|---|---|
| **continued** | **For Translation:**<br>The graph is translated _________(up/down/left/right) _____(#) units.<br><br>**For Reflection:**<br>The graph is reflected over the ________________(x-axis/y-axis). | **For Translation:**<br>The graph is translated ________(up/down/left/right) _____(#) units.<br><br>**For Reflection:**<br>The graph is reflected over the ________________(x-axis/y-axis). | | d. "The graph of f of x equals 1 over x is stretched vertically by a factor of 3." | d. "The graph of f or x equals 1 over x is stretched vertically by a factor of 3." | |

| ELD STANDARD 3: The Language of Mathematics | | | | | | MAISA Algebra 2, Unit 4, Rational Expressions & Functions |
|---|---|---|---|---|---|---|

**EXAMPLE CONTEXT FOR LANGUAGE USAGE:** Students will explain how to use a table or a graph to solve an equation in the form of *f(x) = g(x)* , where at least one of the functions is a rational function. This strand addresses the content standard HSA-REI.D.11: "Explain why the x-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)* ; find the solutions approximately..." Technology plays an important role in this process. Allowing students to generate tables and graphs with technology focuses their attention on analyzing and manipulating the graphs and/or tables in order to find and justify solutions. Note: Although the example language in levels 4 and 5 only illustrates how students might reason using a table, students should have opportunities to create and critique reasoning for multiple representations that also include graphs (and possibly equations for simple cases).

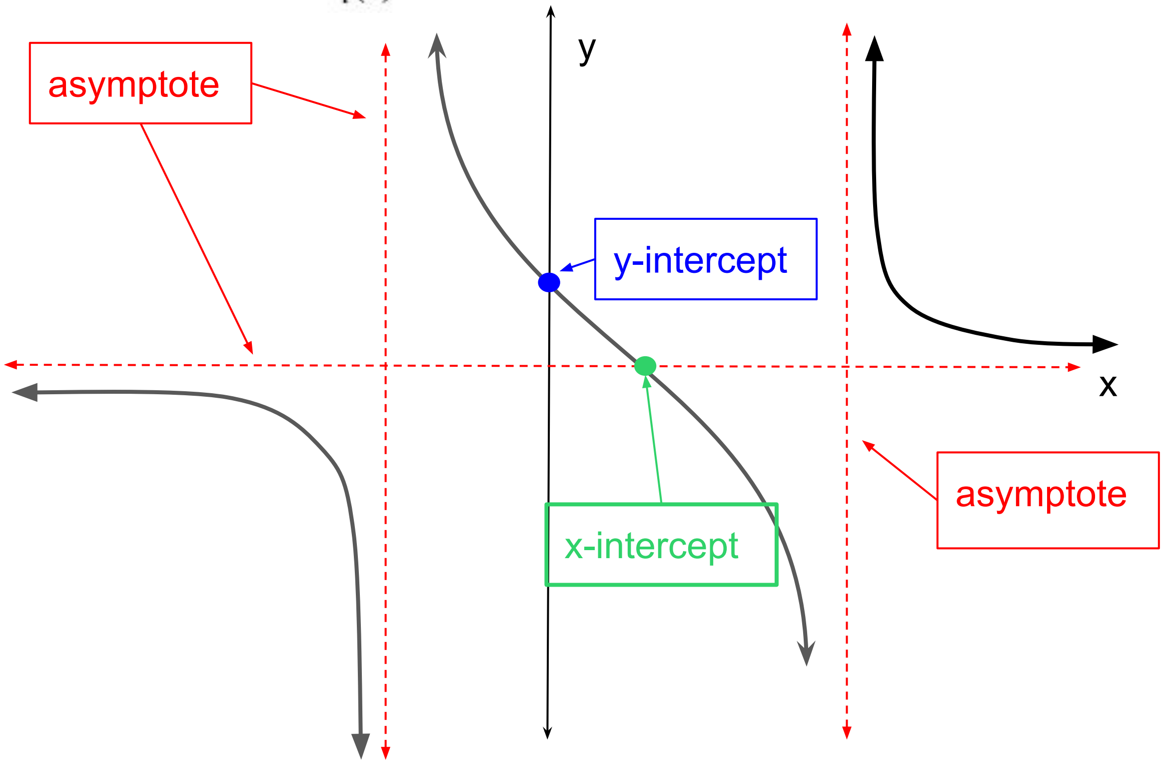
Students at all levels of proficiency may benefit from the opportunity to talk through their thinking with a partner. This allows students oral practice with the language of mathematics as related to systems of rational functions and other functions before producing the written product.

**COGNITIVE FUNCTION:** Students at all levels of English language proficiency analyze an equation containing two functions and **EXPLAIN** a method for solving using a table or a graph.

| | **Level 1<br>Entering** | **Level 2<br>Emerging** | **Level 3<br>Developing** | **Level 4<br>Expanding** | **Level 5<br>Bridging** | **Level 6<br>Reaching** |
|---|---|---|---|---|---|---|
| **Writing** | Explain in words and/or phrases a method for using a table or a graph to solve an equation in the form *f(x) = g(x)* or *f(x)-g(x)=0* in which at least one of the functions is a rational function, using sentence frames with choices and choosing appropriate terms on a function families anchor chart and an illustrated word bank.<br><br>This is a(n) _______ function and a(n)_________ function. (choose from the anchor chart)<br><br>First _________ (write an equivalent equation, graph, make a table).<br><br>Next _________ (write an equivalent equation, graph, make a table).<br><br>I see the solution(s) on the ______ (table/graph). | Explain in simple sentences, accompanied by a student-produced labeled sketch, a method for using a table or a graph to solve an equation in the form *f(x) = g(x)* or *f(x)-g(x)=0* in which at least one of the functions is a rational function, using sentence frames with choices and choosing appropriate terms on a function families anchor chart and an illustrated word bank.<br><br>This is a(n) _______ function and a(n)_________ function. (choose from the anchor chart)<br><br>First _________ (write an equivalent equation, graph, make a table).<br><br>Next _________ (write an equivalent equation, graph, make a table). | Explain in complete sentences a method for using a table or a graph to solve an equation in the form *f(x) = g(x) or f(x)-g(x)=0* in which at least one of the functions is a rational function, referring to a function families anchor chart and using sentence frames/stems with a suggested word list (e.g., equation, function, equivalent equation, table, graph, intersect, coordinates, solutions, approximate, changing signs, zeros).<br><br>This is a(n) _______ function and a(n)_________ function. (choose from the anchor chart)<br>First I would ...<br>Then I would ...<br>Next I would ...<br>The solution(s) to the equation is/are ... | Explain in compound and/or complex sentences a method for using a table or a graph to solve an equation in the form *f(x) = g(x)* or *f(x)-g(x)=0* in which at least one of the functions is a rational function, referring to a function families anchor chart and using a suggested word list (equation, function, rearrange, table, graph, intersect, coordinates, solutions, approximate).<br><br>E.g., [Given: Explain how to use a table or graph to solve the equation *(x-4)(x+5)/(x+4) - x/(x-2) = 0* .]<br><br>"This equation has two rational functions. Because the equation is already equal to zero, I can look at a table and find when the y-values change in sign from | Explain in compound and/or complex sentences a method for using a table or a graph to solve an equation in the form *f(x) = g(x)* or *f(x)-g(x)=0* in which at least one of the functions is a rational function, referring to a function families anchor chart.<br><br>E.g., [Given: Explain how to use a table or graph to solve the equation *(x-4)(x+5)/(x+4) - x/(x-2) = 0* .]<br><br>"The equation has two rational functions. Because the equation is already equal to zero, I can look at a table and find when the *y-* values change in sign from positive to negative and vice versa. Then I can change the table settings to smaller and smaller increments in order to | |

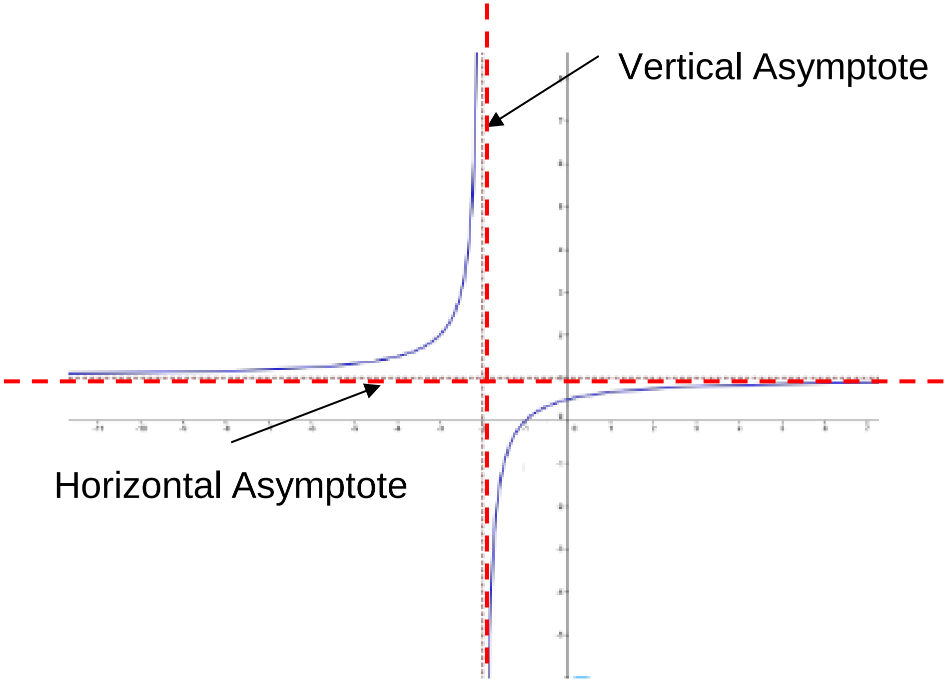
| | Level 1<br>Entering | Level 2<br>Emerging | Level 3<br>Developing | Level 4<br>Expanding | Level 5<br>Bridging | Level 6<br>Reaching |
|---|---|---|---|---|---|---|
| **continued** | The solution(s) is/are _____.<br><br>[Alternatively, students could sketch the representation they are using to solve the equation and provide words or short phrases to label the solution or process.] | I see the solution(s) on the ______ (table/graph).<br><br>The solution(s) is/are _____.<br><br>[Students should sketch the representation they are using to solve the equation and provide words or short phrases to label the solution or process.] | | positive to negative and vice versa. Then I can change the table settings to smaller and smaller increments in order to more closely approximate the x-value that causes the y-value to equal zero. I can use a graph to verify how many solutions there should be. The solution is x = -10." | more closely approximate the x-value that causes the y-value to equal zero. I can use a graph to verify how many solutions there should be. The solution is x = -10." | |

Rational function $R(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials and q(x) ≠ 0.

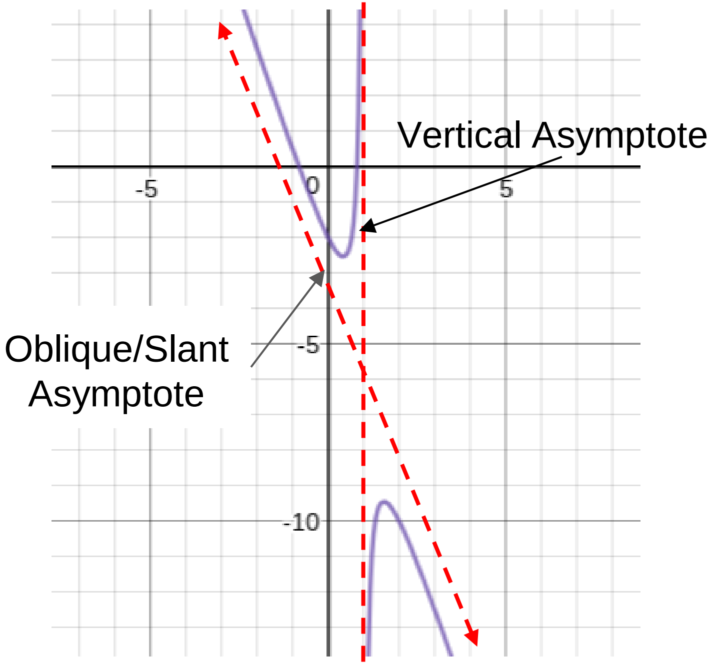


**Asymptote** - Part of a curve goes to +/- infinity. An asymptote is a line close to the function’s graph as the *y* or *x* values go to +/- infinity

## Vertical Asymptote/Horizontal Asymptote




## Oblique/Slant Asymptote



# Polynomial Function

A **polynomial function** is a sum of one or more terms. Each term is the product of a real number and a variable with an exponent. The exponent is a whole number.
**OR** A **polynomial function** is a product of polynomials.

## Examples

$f(x) = 3x + 3$ $g(x) = x^3 - 7x + 4$ $h(x) = (x + 1)(5x - 4)$

## Nonexamples

$$m(x) = 3x^{-2}$$

$$V(x) = \frac{3}{x-4} = 3(x-4)^{-1}$$

$$b(x) = 5x^4 + 3x^2 - 2x^{\frac{1}{5}} - 10$$

Not whole numbers

# Rational Function

Rational function $R(x) = \frac{p(x)}{q(x)}$

where p(x) and q(x) are polynomials and q(x) ≠ 0.

## Examples

$y(x) = \frac{x+1}{x-4}$ $u(x) = \frac{x-2}{x^2 - x - 12}$ $p(x) = \frac{x-2}{(x-4)(x+3)}$

## Nonexamples

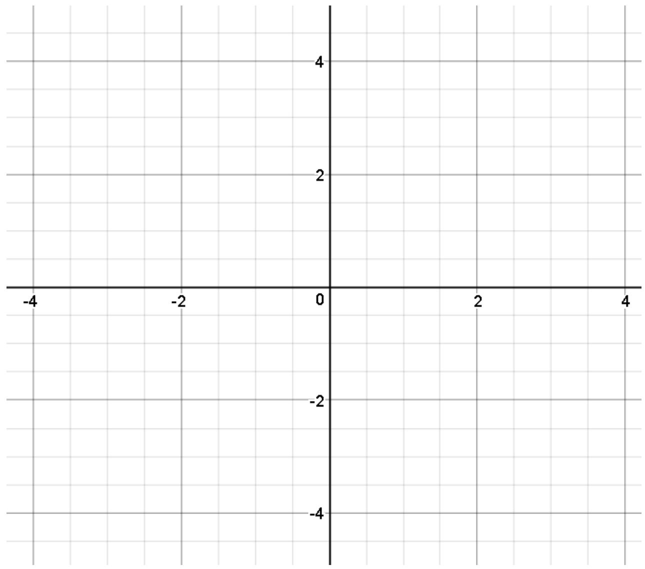
$$S(x) = \frac{2-\sqrt{x}}{4-x} = \frac{2-x^{\frac{1}{2}}}{4-x}$$

$$q(x) = \frac{x-2}{4-\frac{3}{x}} = \frac{x-2}{4-3x^{-1}}$$

$$t(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

Not whole numbers

Using your calculator, graph the parent function $f(x) = \frac{1}{x}$. Sketch the graph on the grid below.




Using your calculator, graph each equation below (one at a time) and explain how the one change in the equation will affect the graph of the parent function $f(x) = \frac{1}{x}$.

a. $f(x) = \frac{1}{x+3}$

Explanation:

b. $f(x) = \frac{1}{x} + 3$

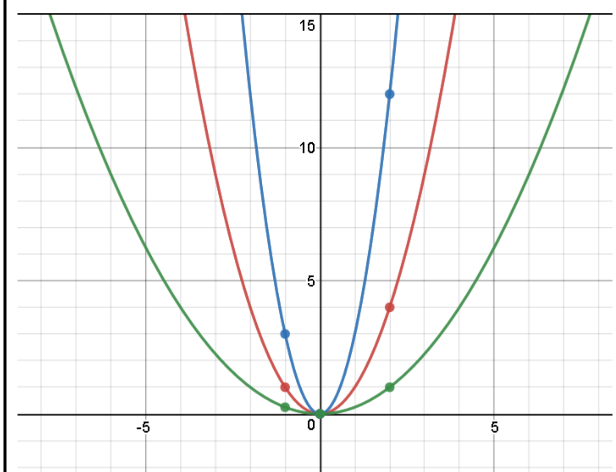
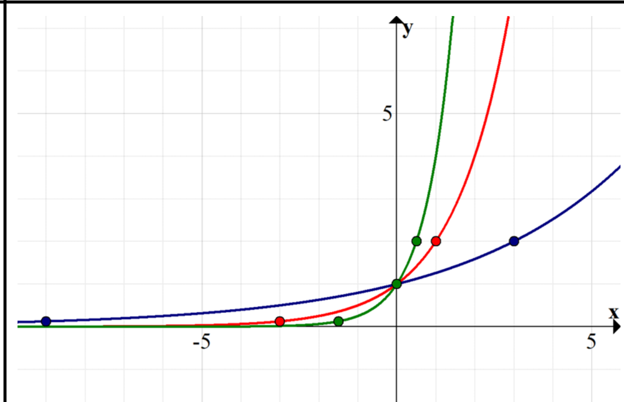
Explanation:

c. $f(x) = \frac{3}{x}$

Explanation:

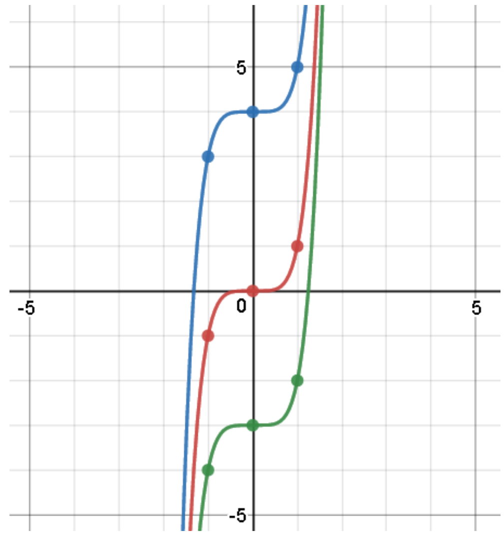
d. $f(x) = \frac{1}{3x}$

Explanation:

<table>
<tr><th>Type of Transformation and details</th><th>Table</th><th>Graph</th></tr>
<tr><td>VERTICAL<br>STRETCH or<br>SHRINK<br>by a factor of...</td><td>
<table>
<tr><th>$x$</th><th>$x^2$</th><th>$3x^2$<br>stretch by 3</th><th>$\frac{x^2}{4}$<br>shrink by 4</th></tr>
<tr><td>-1</td><td>1</td><td>3</td><td>0.25</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>2</td><td>4</td><td>12</td><td>1</td></tr>
</table>
</td><td>
</td></tr>
<tr><td>HORIZONTAL<br>STRETCH or<br>SHRINK<br>by a factor of...</td><td>
<table>
<tr><th>$y$</th><th>x value for<br>$2^x$</th><th>x value for<br>$(2)^{\left(\frac{x}{3}\right)}$<br>stretch by 3</th><th>x value for<br>$2^{2x}$<br>shrink by 2</th></tr>
<tr><td>0.125</td><td>-3</td><td>-9</td><td>-1.5</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>2</td><td>1</td><td>3</td><td>0.5</td></tr>
</table>
</td><td>
</td></tr>
</table>

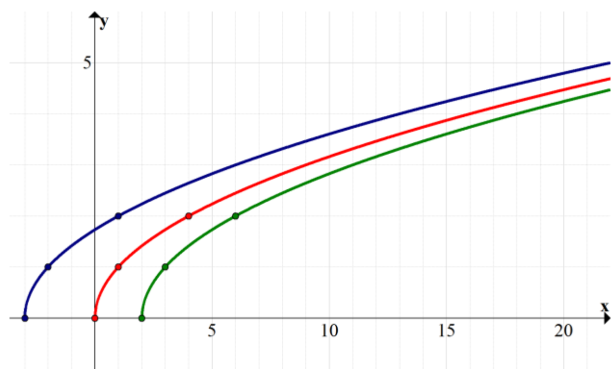
**VERTICAL TRANSLATION**
***up*** **or** ***down___units***

| $x$ | $x^5$ | $x^5+4$ *up 4* | $x^5-3$ *down 3* |
|---|---|---|---|
| -1 | -1 | 3 | -4 |
| 0 | 0 | 4 | -3 |
| 1 | 1 | 5 | -2 |



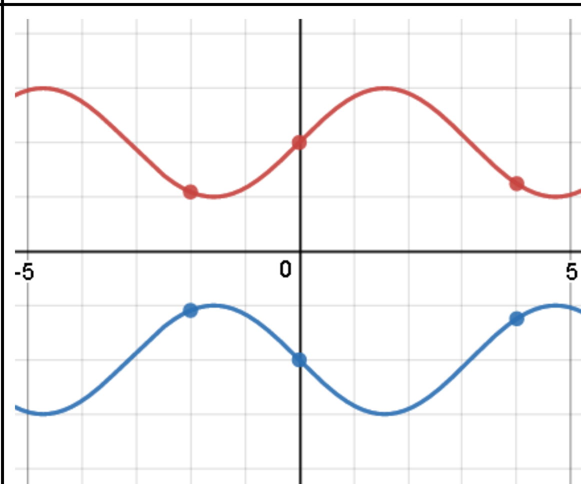

**HORIZONTAL TRANSLATION**
***left*** **or** ***right___units***

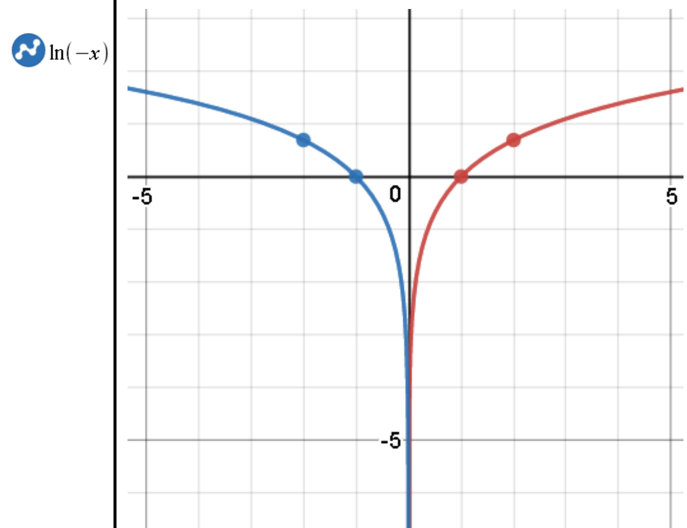
| *y* | $\sqrt{x}$ | $\sqrt{x+3}$ *left 3* | $\sqrt{x-2}$ *right 2* |
|---|---|---|---|
| 0 | 0 | -3 | 2 |
| 1 | 1 | -2 | 3 |
| 2 | 4 | 1 | 6 |




**REFLECTION**
***over x-axis***

| $x$ | $\sin x+2$ | $-(\sin x+2)$ *over* |
|---|---|---|
| -2 | 1.09 | -1.09 |
| 0 | 2 | -2 |
| 4 | 1.24 | -1.24 |

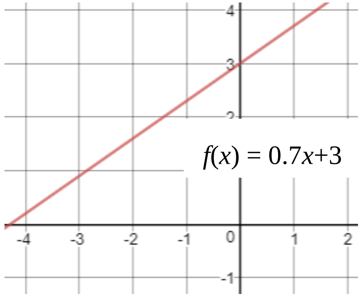


<table>
<tr><td></td><td></td><td></td></tr>
<tr>
<td>REFLECTION<br>over y-axis</td>
<td>
<table>
<tr><td>x-value for $\ln x$</td><td>y</td><td>x-value for $\ln -x$ over y-axis</td></tr>
<tr><td>0</td><td>undefined</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>-1</td></tr>
<tr><td>2</td><td>0.69</td><td>-2</td></tr>
</table>
</td>
<td>
</td>
</tr>
</table>

# Function Families $a \neq 0$

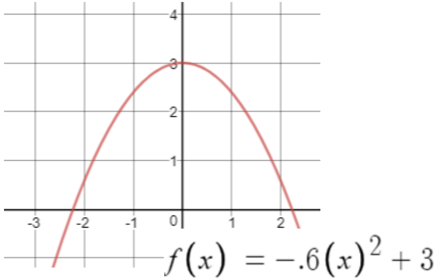
## Linear Function

$f(x) = a(x-h)+k$



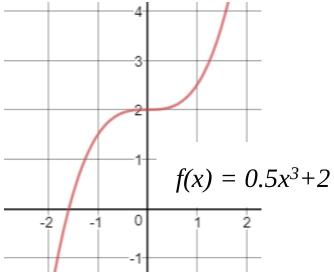

## Quadratic Function

$f(x) = a(x-h)^2 + k$



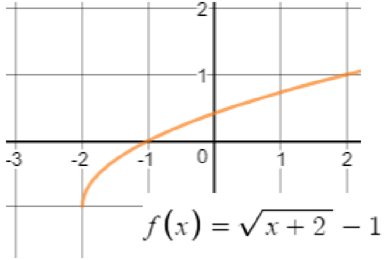

## Cubic Function

$f(x) = a(x-h)^3 + k$



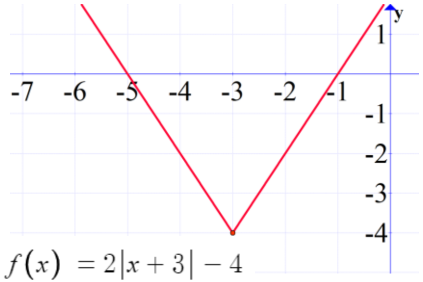

## Square Root Function

$f(x) = a\sqrt{x-h} + k$



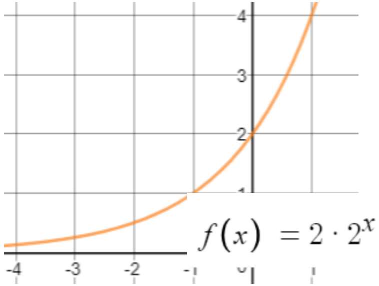

## Absolute Value Function

$f(x) = a|x-h| + k$



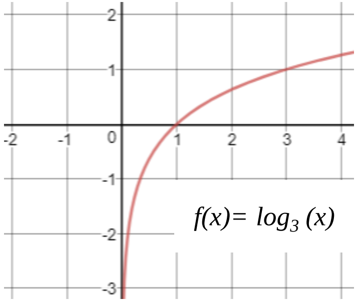

## Exponential Function

$f(x) = a(b)^x$



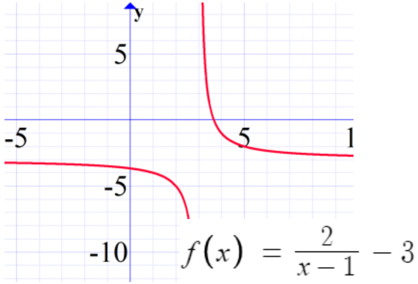

## Logarithmic Function

$f(x) = a\log_b(x)$




## Rational Function

$f(x) = \frac{a}{x-h} + k$



## Equation

$$3x + 2y = 18$$

$$\frac{2}{x+1} = x \qquad \frac{2x}{3x-2} = 0$$

## Write Equivalent Equations

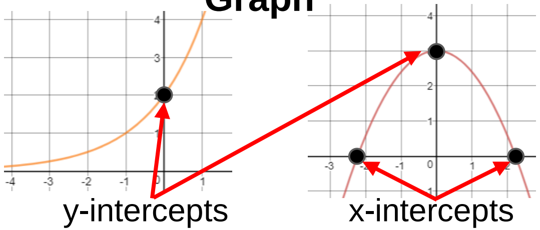
(see "Solving Equations")

$$\frac{2}{x+1} = x$$
$$2 = (x)(x+1)$$
$$2 = x^2 + x$$
$$0 = x^2 + x - 2$$
$$0 = (x-1)(x+2)$$
$$x = 1, x = -2$$

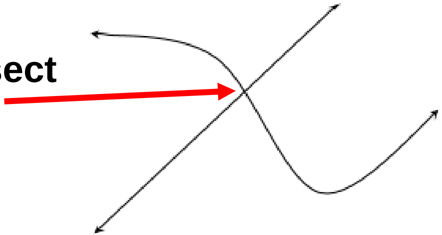
## Table

| *x* | *y* |
|---|---|
| -2 | -4 |
| 0 | 0 |
| 2 | 4 |
| 4 | 8 |

## Graph




## Intersect



## Coordinates

( -3, 9)

x-coordinate

y-coordinate

## Solution

$$21 + 3x = 27$$
$$-21 \qquad -21$$
$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$
$$21 + 3(2) = 27 \checkmark$$

## Approximate

*25/7 is 3.57142857…, which is about 3.6*

# Solving Equations

**Write Equivalent Equations:**

$$\frac{2}{x+1} = x$$

$$2 = (x)(x+1)$$

$$2 = x^2 + x$$

$$0 = x^2 + x - 2$$

$$0 = (x-1)(x+2)$$

$$x = 1, x = -2$$

## Equivalent Equations Share the Same Solutions

The solutions are x = 1, and x = -2

**Check that each equation has the same solution:**

$$\frac{2}{1+1} = \frac{2}{2} = 1 \checkmark$$

$$(1)(1+1) = (1)(2) = 2 \checkmark$$

$$(1)^2 + (1) = 2 = 2 \checkmark$$

$$(1)^2 + (1) - 2 = 2 - 2 = 0 \checkmark$$

$$(1-1)(1+2) = (0)(3) = 3 \checkmark$$

$$1 = 1 \checkmark$$

$$\frac{2}{-2+1} = \frac{2}{-1} = -2 \checkmark$$

$$(-2)(-2+1) = (-2)(-1) = 2 \checkmark$$

$$(-2)^2 + (-2) = 2 = 2 \checkmark$$

$$(-2)^2 + (-2) - 2 = 4 - 4 = 0 \checkmark$$

$$(-2-1)(-2+2) = (-3)(0) = 0 \checkmark$$

$$-2 = -2 \checkmark$$