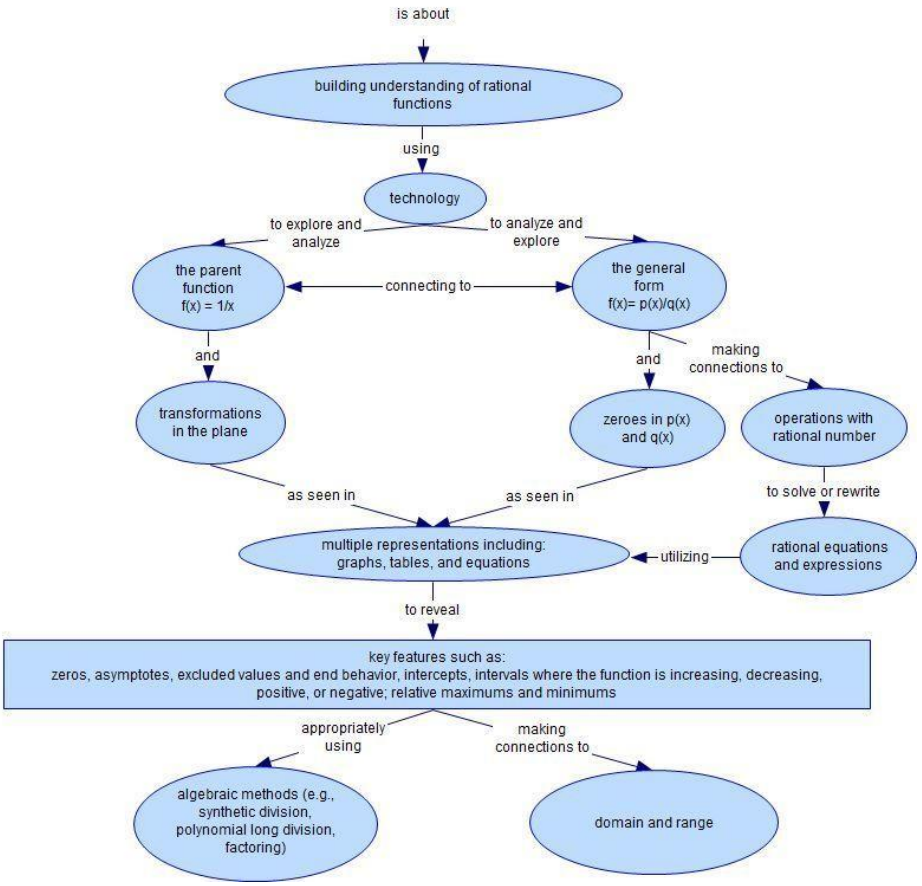
Common Core Initiative

Overarching Questions and Enduring Understandings

How does understanding polynomial functions (and other function families) aid in making sense of rational functions?

Graphic Organizer



Unit Abstract

This unit introduces students to the family of rational functions by building on knowledge of linear, quadratic, and polynomial functions studied extensively in previous units. These familiar function families are used as expressions and combined using the four arithmetic operations in a way that is similar to how integers are combined to form rational numbers. This leads to a general form:
f(x)=g(x)/h(x), where *g(x)* and *h(x)* are polynomial functions. The study of rational functions also creates opportunities for students to engage with beginning concepts of limits, that will be studied in future courses, as they examine the end behavior and behavior near asymptotes.

Because of the content and algebraic complexity in this unit, technology plays a vital role in allowing students to investigate key features of rational functions and the relationships between representations. Technology opens

access to concepts early in the learning trajectory and facilitates problem solving allowing more students to make connections and reason about this new function family. For example, technology allows students to efficiently create and move between various representations in a fraction of the time it would take to generate the representations with paper and pencil. The development of a deep understanding of function classes and their characteristics as called for in the CCSS-M's function domain requires the effective use of technology in teaching and learning.

In a functions-based approach to algebra, each family begins with a parent function, in this case f(x)=1/x, that can be transformed into other members of the function family by "replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative)" (CCSS-M, HSF-BF.B.3) In the case of rational functions, *k* may also take on the form of a polynomial function. Students should examine and discuss the effect of these changes (e.g. multiplying by a factor of 1/(1x+2) would, in effect, create another asymptote and change the end behavior in a similar way to that of a polynomial function moving from odd to even degree).

These two algebraic forms of rational functions (i.e., the general form, f(x)=g(x)/h(x), and the parent function, f(x)=1/x) are useful in different situations. The general form is useful for many applications (e.g. forming a "rate" with two functions, one representing profit based on number of attendees for an event and one representing ticket price based on number of attendees, with the rate function representing profit per ticket) while the parent function is useful when examining ideas of transformation in the plane, and algebraic relationships between different rational functions. Students should experience both during this unit and be provided with multiple opportunities to use and make connections among graphic, tabular, and symbolic representations of rational functions.
In this unit, students also learn to operate on rational expressions and relate to experiences with rational operations. The analogous set of properties with rational numbers provides the rationale for the use of synthetic division and polynomial long division. Operations on rational expressions are presented as tools to better understand features and behaviors of rational functions (e.g. zeros, asymptotes, and end behaviors). Students should have experiences that drive home these ideas and foster both a need for and understandings of procedures.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Algebra

Arithmetic with Polynomials & Rational Functions

HSA-APR.B. Understand the relationship between zeros and factors of polynomials.

- HSA-APR.B.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

HSA-APR.D. Rewrite rational expressions.

- HSA-APR.D.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- HSA-APR.D.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

The Real Number System

HSN-RN.B. Use properties of rational and irrational numbers and that the product of a nonzero rational number and an irrational number is irrational.

- HSN-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

MI: HS: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

- HSA-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations & Inequalities

HSA-REI.A. Understand solving equations as a process of reasoning and explain the reasoning.

- HSA-REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

HSA-REI.D. Represent and solve [rational] equations and inequalities graphically.

- HSA-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Developed and/or Utilized Across Multiple Units

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.2. Create [rational] equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Interpreting Functions
HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Building Functions
HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Interpreting Functions
HSF-IF.B. Interpret functions that arise in applications in terms of the context.

	• HSF-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★
Essential/Focus Questions 1. How can equations and tables of values for rational functions help reveal key features their graphs? 2. How can the key features of graphs of rational functions be used to create an algebraic model? 3. How do different forms of rational functions highlight structures where polynomial functions or transformations can aid in analyzing rational functions? 4. (+) How does understanding operations with rational numbers inform operations with rational expressions?	**Key Concepts** asymptote (horizontal, vertical, slant) continuity (continuous, discontinuous, holes/undefined points) domain and range end behavior intercepts (x-intercept, y-intercept) rational function solutions to rational equations (extraneous solutions and solutions)
Assessment Tasks Assessment Overview Independent Practice Assessment Task	**Intellectual Processes** **Standards for Mathematical Practice** *Students will have opportunities to:* • **construct viable arguments and critique the reasoning of others** by analyzing and using multiple representations of rational functions*;* • **look for and make use of structure** within rational functions (represented in multiple ways) by making connections to knowledge of other families of functions (such as polynomial functions); • **reason abstractly and quantitatively** within mathematical and real world contexts involving rational functions, connecting operations with rational numbers to operations with rational functions, and transferring meaning between multiple representations of rational functions; and • **use appropriate tools strategically** while working with rational functions by making thoughtful decisions about when to use technology and taking into account its limitations in representation.
Lesson Sequence Lesson Overview Student Handout Professional Learning Task – Student Work Samples: Reengagement of the Lesson	**Resources** Unit Resources