ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 5, Sequences & Series

CONNECTIONS: Michigan Academic State Standards for Mathematics

EXAMPLE CONTEXT FOR LANGUAGE USAGE: This strand addresses the mathematics standard HSA-SSE.B.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. In this sample task from Illustrative Mathematics, students will use the real-life context of a "YouTube Explosion" to figure out how many people will have seen a video after a certain number of successive shares, and then derive a formula for a finite geometric series based on that context. In the task as originally described, teachers are encouraged to have students work in small groups and also to have the small groups share their ideas and findings with the whole class as they work together to derive the formula. The strand below is written for students to engage with the context and make sense of what they hear. As students listen to the context, if they are not provided with a visual representation, they should be encouraged to sketch their own visual representation (tree diagram, etc.).

Due to the linguistic complexity of the task, it may be beneficial for students with the same first language to work together in cooperative groups. This allows students to problem solve in a language in which they are most comfortable as well as allows students to find common cognates or other language features that unlock the meaning of the concepts in English. The mathematical demand of the task also adds complexity. As such, it may assist students to have written copy of the task in front of them as they listen. (Generally, a printed version is not given when students are building proficiency with listening so as not to rely on a written form.)

This strand is written to demonstrate how students might listen in a whole group launch of the task introducing the context (video explosion) and the initial problem-solving tool of a table. After the initial launch of the task, students would work in small groups collaboratively reading the remaining questions and discussing their problems solving. Additional scaffolds would be helpful for this dynamic.

Link to the task at Illustrative Mathematics website: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/4/tasks/1797. Students should have additional opportunities for listening and speaking when the class comes back together to discuss both individual parts of the task and the task as a whole.

COGNITIVE FUNCTION: Students at all levels of English language proficiency will **INTERPRET** oral instructions in order to **DERIVE** a mathematical formula in a real-life context.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Interpret instructions read aloud multiple times, with purposeful pauses and gestures, pointing to visual representations, for deriving a mathematical formula in a real life context, using a student- or teacher-created reference sheet and a visual representation of the context, while working in a small group with students with the same first languge and/or students of higher English language proficiency. E.g., Michelle, Hillary, and Cory [pause and point]	Interpret instructions read aloud multiple times, with purposeful pauses and gestures, pointing to visual representations, for deriving a mathematical formula in a real life context, using a student- or teacher-created reference sheet and a visual representation of the context, while working in a small group of with students with the same first languge and/or studentswith students of higher English language proficiency. E.g., Michelle, Hillary, and Cory [pause and point] created a YouTube video	Interpret a real life mathematical context read aloud with purposeful pauses and repeating as necessary in order to derive a mathematical formula using a student- or teacher-created reference sheet, a visual representation of the context, and working in a small group. E.g., Michelle, Hillary, and Cory [pause] created a YouTube video [pause], and have a plan to get as many people to watch it as possible [pause]. They will each share the video with 3 of their best	Interpret a real life mathematical context read aloud with purposeful pauses in order to derive a mathematical formula, using a student- or teacher-created reference sheet and working in a small group. E.g., Michelle, Hillary, and Cory [pause] created a YouTube video [pause], and have a plan to get as many people to watch it as possible [pause]. They will each share the video with 3 of their best friends	Interpret a real life mathematical context read aloud with purposeful pauses in order to derive a mathematical formula, using a student- or teacher-created reference sheet and working in a small group. E.g, Michelle, Hillary, and Cory [pause] created a YouTube video [pause], and have a plan to get as many people to watch it as possible [pause]. They will each share the video with 3 of their best friends	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
	created a YouTube video [pause and point], and have a plan to get as many people to watch it as possible [pause]. They will each share the video with 3 of their best friends [pause and point], and create a caption on the video [pause and point] that says, "Please share this video with 3 of your best friends." [pause and point] Each time the video is shared with someone [pause], that person instantly views the video only once [pause and hold up one finger] and sends it to exactly 3 more people [pause and point]. In addition, assume that every person only receives the video once [pause and hold up one finger]. Suppose that after 1 hour since the video was posted [pause and hold up one finger], Michelle, Hillary, and Cory have watched the video [pause and point]; after 2 hours [pause and hold up two fingers], all of their friends have watched the video [pause and point]; and so on [pause]. a) Fill in the table [pause] for the number of people who received the video during the given hour [pause] .... [The link to the entire activity is provided above in the Example Context.]	[pause and point], and have a plan to get as many people to watch it as possible [pause]. They will each share the video with 3 of their best friends [pause and point], and create a caption on the video [pause and point] that says, "Please share this video with 3 of your best friends." [pause and point] Each time the video is shared with someone [pause], that person instantly views the video only once [pause and hold up one finger] and sends it to exactly 3 more people [pause and point]. In addition, assume that every person only receives the video once [pause and hold up one finger]. Suppose that after 1 hour since the video was posted [pause and hold up one finger], Michelle, Hillary, and Cory have watched the video [pause and point]; after 2 hours [pause and hold up two fingers], all of their friends have watched the video [pause and point]; and so on [pause]. a) Fill in the table [pause] for the number of people who received the video during the given hour [pause] .... [The link to the entire activity is provided above in the Example Context.]	friends [pause], and create a caption on the video [pause] that says, "Please share this video with 3 of your best friends." [pause] Each time the video is shared with someone [pause], that person instantly views the video only once [pause] and sends it to exactly 3 more people [pause]. In addition, assume that every person only receives the video once [pause]. Suppose that after 1 hour since the video was posted [pause], Michelle, Hillary, and Cory have watched the video [pause]; after 2 hours, all of their friends have watched the video; and so on [pause]. a) Fill in the table [pause] for the number of people who received the video during the given hour [pause] .... [The link to the entire activity is provided above in the Example Context.]	[pause], and create a caption on the video [pause] that says, "Please share this video with 3 of your best friends." [pause] Each time the video is shared with someone [pause], that person instantly views the video only once [pause] and sends it to exactly 3 more people [pause]. In addition, assume that every person only receives the video once [pause]. Suppose that after 1 hour since the video was posted [pause], Michelle, Hillary, and Cory have watched the video [pause]; after 2 hours, all of their friends have watched the video; and so on [pause]. a) Fill in the table [pause] for the number of people who received the video during the given hour [pause] .... [The link to the entire activity is provided above in the Example Context.]	[pause], and create a caption on the video [pause] that says, "Please share this video with 3 of your best friends." [pause] Each time the video is shared with someone [pause], that person instantly views the video only once [pause] and sends it to exactly 3 more people [pause]. In addition, assume that every person only receives the video once [pause]. Suppose that after 1 hour since the video was posted [pause], Michelle, Hillary, and Cory have watched the video [pause]; after 2 hours, all of their friends have watched the video; and so on [pause]. a) Fill in the table [pause] for the number of people who received the video during the given hour [pause] .... [The link to the entire activity is provided above in the Example Context.]	

ELD STANDARD 3: The Language of Mathematics **MAISA Algebra 2, Unit 5, Sequences & Series**

EXAMPLE CONTEXT FOR LANGUAGE USAGE: This strand addresses standard HSF-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. The focus of the sample task is on translating between the two forms. Students will need to recognize a given rule as recursive or explicit, identify the first term and the common ratio or difference, and use those properties to write the other form of the rule. This task could be speaking or writing, as students explain the process of identifying the key characteristics of the sequence and applying them to writing a rule. To further support the needs of Level 1 students, the teacher might assist the student in highlighting words/concepts from the sentence frames that correspond to the illustrated reference sheet (e.g., previous term). This is an additional visual scaffold for students at the lowest levels of language proficiency. Additionally, students at lower levels of proficiency will benefit from hearing students at the higher levels of proficiency modeling language usage in their answers first.

Mathematically, this task can be more challenging when an explicit form has been simplified and the first terms of either arithmetic or geometric sequences are not easily identified thus requiring algebraic manipulation or evaluating to convert from explicit to recursive form. (For example, an explicit arithmetic formula can be written as $a_n = 5 + 6(n-1)$ or $a_n = -1 + 6n$ and an explicit geometric formula can be written as $a_n = 24\ (6)^{(n-1)}$ or $a_n = 4\ (6^n)$. For both, if the second form is given, students will need to manipulate it back to the standard explicit form to find the initial term, or create a table of values and use patterns or evaluating the equation to project back to the initial term.)

COGNITIVE FUNCTION: Students at all levels of English language proficiency apply knowledge of arithmetic and geometric sequences to **EXPLAIN** the process of translating between explicit and recursive formulas.

	Level 1 **Entering**	**Level 2** **Emerging**	**Level 3** **Developing**	**Level 4** **Expanding**	**Level 5** **Bridging**	**Level 6** **Reaching**
Speaking	Explain to a partner in short sentences or phrases the process of translating between the explicit and recursive forms of arithmetic and/or geometric sequences using an illustrated reference sheet and sentence frames with choices. This sequence has a common ________ (difference/ratio). That means it is _________ (arithmetic/geometric). The first term is ____ (#). The common ________(difference/ratio) is ___ (#). Each term is ____ (#) _________ (times/plus) the previous term. The ________ (explicit/recursive) rule is______[say the equation].	Explain to a partner in short sentences or phrases the process of translating between the explicit and recursive forms of arithmetic and/or geometric sequences using an illustrated reference sheet and sentence frames with choices. This sequence has a common ________ (difference/ratio). That means it is _________ (arithmetic/geometric). The first term is ____ (#) The common ________(difference/ratio) is ___ (#). Each term is ____ (#) _________ (times/plus) the previous term. The ________ (explicit/recursive) rule is_______[say the equation].	Explain to a partner in complete sentences the process of translating between the explicit and recursive forms of arithmetic and/or geometric sequences using an illustrated reference sheet and a suggested word list, (e.g., recursive, explicit, sequence, arithmetic/geometric, common ratio/difference, term, initial, previous). [Sample Prompt A: Explain how to write a recursive rule for the sequence with the explicit rule $a_n = 3(0.5)^{n-1}$] Sample Student Response A: “The explicit rule is $a_n = 3(0.5)^{n-1}$. This equation has a common ratio. That means this is a geometric sequence. The initial term is 3 and the common ratio is 0.5. The recursive rule needs the initial	Explain to a partner in compound and/or complex sentences, with appropriate transition words, the process of translating between the explicit and recursive forms of arithmetic and/or geometric sequences using a suggested word list, (e.g., recursive, explicit, sequence, arithmetic/geometric, common ratio/difference, term, initial, previous). [Sample Prompt A: Explain how to write a recursive rule for the sequence with the explicit rule $a_n = 3(0.5)^{n-1}$] Sample Student Response A: “If the explicit rule is $a_n = 3(0.5)^{n-1}$, there is a common ratio which tells	Explain to a partner in compound and/or complex sentences, with appropriate transition words, the process of translating between the explicit and recursive forms of arithmetic and/or geometric sequences using a suggested word list, (e.g., recursive, explicit, sequence, arithmetic/geometric, common ratio/difference, term, initial, previous.) [Sample Prompt A: Explain how to write a recursive rule for the sequence with the explicit rule $a_n = 3(0.5)^{n-1}$.] Sample Student Response A: “If the explicit rule is $a_n = 3(0.5)^{n-1}$, there is a common ratio which tells me this is a geometric	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
			term $a_1 = 3$. Each term is 0.5 times the previous term. So $a_n = 0.5*a_{n1}$." [Sample Prompt B: Explain how to write an explicit rule for the sequence with the recursive rule $a_1 = 25$ and $a_n = a_{n-1} + 4$.] Sample Student Response B: "The equation $a_1 = 25$ means the initial term is 25. The equation $a_n = a_{n-1} + 4$ means that each term is 4 plus the previous term. So this is an arithmetic sequence with a common difference of 4. The equation for an arithmetic sequence is $a_n = a_1 + (n-1)d$. So the explicit rule is $a_n = 25 + 4(n-1)$. This simplifies to $a_n = 4n + 21$."	me this is a geometric sequence with the initial term equal to 3 and the common ratio of 0.5. To write the recursive rule, I need to state the initial term and then the rule for each term based on the previous term. The initial term is $a_1 = 3$, and each term is 0.5 times the previous term, so $a_n = 0.5*a_{n-1}$." [Sample Prompt B: Explain how to write an explicit rule for the sequence with the recursive rule $a_1 = 25$ and $a_n = a_{n-1} + 4$.] Sample Student Response B: "The equation $a_1 = 25$ means the initial term is 25. The equation $a_n = a_{n-1} + 4$ means that each term is 4 plus the previous term. Because we add the same amount each time, this is an arithmetic sequence with a common difference of 4. The equation for an arithmetic sequence is $a_n = a_1 + (n-1)d$. Because $a_1 = 25$ and $d = 4$, the explicit rule is $a_n = 25 + 4(n-1)$. This simplifies to $a_n = 4n + 21$."	sequence with the initial term equal to 3 and the common ratio of 0.5. To write the recursive rule, I need to state the initial term and then the rule for each term based on the previous term. The initial term is $a_1 = 3$, and each term is 0.5 times the previous term, so $a_n = 0.5*a_{n-1}$." [Sample Prompt B: Explain how to write an explicit rule for the sequence with the recursive rule $a_1 = 25$ and $a_n = a_{n-1} + 4$.] Sample Student Response B: "The equation $a_1 = 25$ means the initial term is 25. The equation $a_n = a_{n-1} + 4$ means that each term is 4 plus the previous term. Because we add the same amount each time, this is an arithmetic sequence with a common difference of 4. The equation for an arithmetic sequence is an = a1 + (n-1)d. Because $a_1 = 25$ and $d = 4$, the explicit rule is $a_n = 25 + 4(n-1)$. This simplifies to $a_n = 4n + 21$."	

ELD STANDARD 3: The Language of Mathematics						MAISA Algebra 2, Unit 5, Sequences & Series

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students read, interpret, and analyze a complex mathematical task in order to answer questions about a sequence. This task, from Illustrative Mathematics, - https://www.illustrativemathematics.org/content-standards/HSF/BF/A/2/tasks/1695 gives students the opportunity to engage with concepts related to the standard HSF-IF.A.3. A link to the animation of the problem is located at http://i.imgur.com/dAtcCfH.gif. Sharing the animation with students prior to engaging in the task gives students the opportunity to make sense of the context before processing the language in writing and beginning problem solving.

A student copy of the task is available at the website listed above. A glossed copy, and a simplified version are in the supports. The glossed version is a sample support that was developed with a small group and is representative of what a math teacher would do in the classroom to differentiate.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** and **INTERPRET** a linguistically complex written mathematical description in order to answer questions about a sequence.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading	Analyze a simplified mathematical text in order to answer questions about a sequence using an illustrated reference sheet for contextual and mathematical words and working with a partner. A simplified student copy is found in the supports for this unit.	Analyze a simplified mathematical text in order to answer questions about a sequence using an illustrated reference sheet for contextual and mathematical words and working with a partner. A simplified student copy is found in the supports for this unit.	Analyze a glossed, linguistically complex mathematical text in order to answer questions about a sequence while working with a partner. A glossed student copy is found in the supports for this unit.	Analyze a linguistically complex mathematical text in order to answer questions about a sequence while working with a partner.	Analyze a linguistically complex mathematical text in order to answer questions about a sequence while working with a partner.	

Arithmetic Sequence - **Explicit**

$a_1=3+(1-1)(2)=3$, $a_2=3+(2-1)(2)=5$, $a_3=3+(3-1)(2)=7$, $a_4=3+(4-1)(2)=9, \ldots$

$$\mathbf{a_n = 3 + 2(n-1)}$$

$a_1 = a_1 + 0d$, $a_1 + 1d$, $a_1 + 2d$, $a_1 + 3d, \ldots$

Explicit formula for the n^th^ term: $a_n = a_1 + (n-1)d$

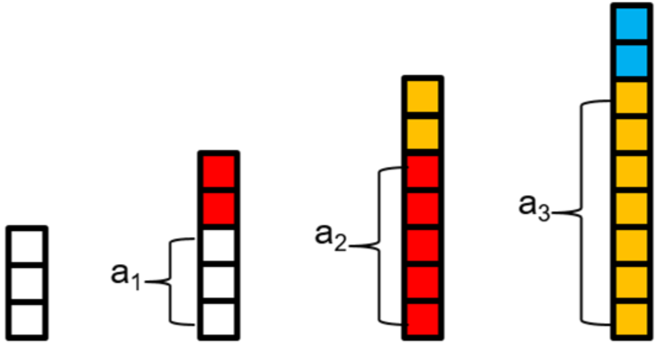
a_1 = initial term

a_n = nth term of the sequence

d = common difference

n = number of the term

Arithmetic Sequence - **Recursive**



$a_1 = 3$, $a_2 = 3 + 2 = 5$, $a_3 = 5 + 2 = 7$, $a_4 = 7 + 2 = 9, \ldots$

$$\mathbf{a_n = a_{n-1} + 2}$$

$a_1 = a_1$, $a_2 = a_1 + d$, $a_3 = a_2 + d$, $a_4 = a_3 + d, \ldots$

Recursive rule for the n^th^ term: $a_n = a_{n-1} + d$

a_1 = initial term

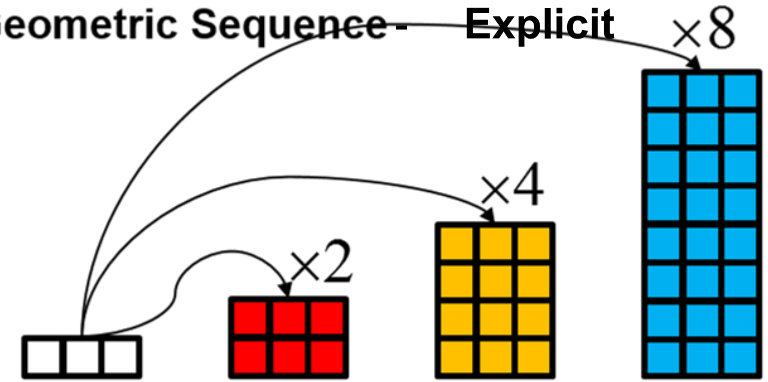
a_n = nth term of the sequence

d = common difference

a_{n-1} = term previous to a_n

n = number of the term

Geometric Sequence - Explicit



$a_1 = 3(2)^{1-1} = 3$, $a_2 = 3(2)^{2-1} = 6$, $a_3 = 3(2)^{3-1} = 12$, $a_4 = 3(2)^{4-1} = 24, \ldots$

$$\mathbf{a_n = 3(2)^{n-1}}$$

$a_1(r)^{1-1}$, $a_1(r)^{2-1}$, $a_1(r)^{3-1}$, $a_1(r)^{4-1}, \ldots$

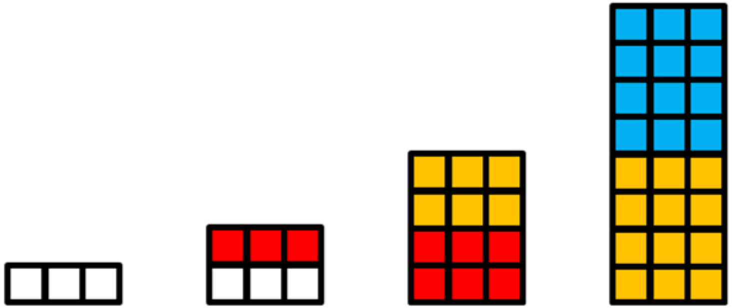
Explicit formula for the n^{th} term: $a_n = a_1(r)^{n-1}$

a_1 = initial term a_n = nth term of the sequence

r = common ratio

n = number of the term

Geometric Sequence - Recursive



$a_1 = 3$, $a_2 = 3(2) = 6$, $a_3 = 6(2) = 12$, $a_4 = 12(2) = 24, \ldots$

$$\mathbf{a_n = a_{n-1}(2)}$$

$a_1 = a_1$, $a_2 = a_1(r)$, $a_3 = a_2(r)$, $a_4 = a_3(r), \ldots$

Recursive rule for the n^{th} term: $a_n = a_{n-1}(r)$

a_1 = initial term a_n = nth term of the sequence

r = common ratio a_{n-1} = term previous to a_n

n = number of the term

Arithmetic Series

1 + 4 + 7 + 10 + . . .

1 + 1+3 + 1+2(3) + 1+3(3) + . . .

a_1 + a_1+1d + a_1+2d + a_1+3d + . . .

Summation Notation:

Finite Series: $S_n = \sum_{i=1}^{n} a_i$ Infinite Series: $S = \sum_{i=1}^{\infty} a_i$

Sum of a finite series: (also called a partial sum) $S_n = \dfrac{n(2a_1 + d(n-1))}{2}$

OR:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right), \quad a_n = a_1 + d(n-1)$$

Geometric Series

1 + 4 + 16 + 64 + . . .

1 + 1(4) + $1(4)^2$ + $1(4)^3$ + . . .

a_1 + $a_1(r)^1$ + $a_1(r)^2$ + $a_1(r)^3$ + . . .

Summation Notation:

Finite Series: $S_n = \sum_{i=1}^{n} a_i$ Infinite Series: $S = \sum_{i=1}^{\infty} a_i$

Sum of a finite series: (partial sum) $S_n = a_1\left(\dfrac{1-r^n}{1-r}\right);\ r \neq 1$

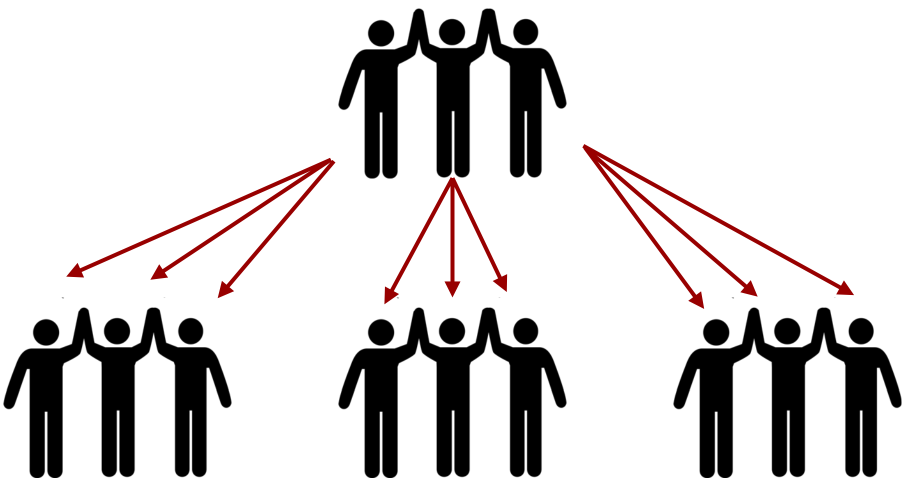
Sum of an infinite series: $S = \left(\dfrac{a_1}{1-r}\right);\ |r| < 1$

"YouTube Explosion" https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/4/tasks/1797

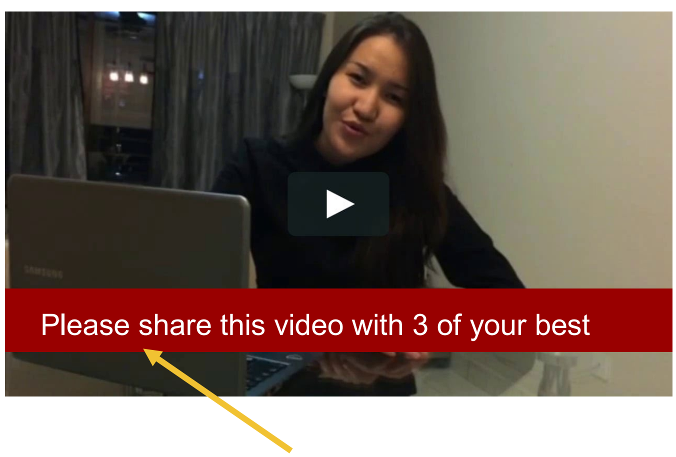
Michelle, Hillary, and Cory

YouTube Video

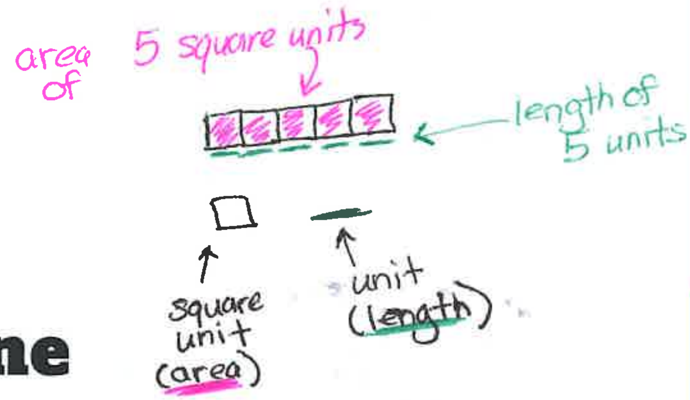
1st Hour: Michelle, Hillary, and Cory "received" the video



2nd hour: Each share the video with 3 of their best friends



A caption on the video



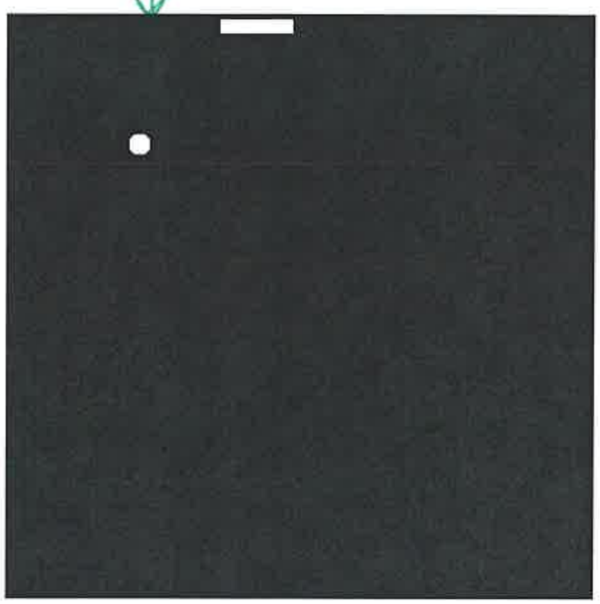
F-IF Snake on a Plane

Task

In a video game called Snake, a player moves a snake through a square ~~region~~ in the plane, trying to eat the white pellets that appear.

balls show up

Flat surface



square units

~~If we imagine~~ the playing field ~~as~~ is a 32-by-32 grid of pixels, then the snake starts as a 4-by-1 rectangle of pixels, and grows in length as it eats the pellets:

square units balls

- After the first pellet (ball), it grows in length by one ~~pixel.~~ unit
- After the second pellet (ball), it further grows in length by two ~~pixels.~~ units
- After the third pellet (ball), it further grows in length by three ~~pixels.~~ units

grows again

units

- and so on, with the n-th pellet increasing its length by ~~n pixels~~.

Let $L(n)$ denote the length of the snake after eating n pellets. For example, $L(3) = 10$.

a. How long is the snake after eating 4 ~~pellets~~ balls? After 5 ~~pellets~~ balls? After 6 pellets balls?

n	4	5	6
L(n)			

b. Find a recursive description of the function $L(n)$. L(0) = ______ L(n) = ______ Rule

Rule for finding the NEXT term from one BEFORE

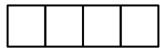
c. Find a non-recursive expression for $L(100)$, and evaluate that expression to compute $L(100)$. L(100) = ______ expression L(100) = ______ #(evaluate)

d. What is the largest number of ~~pellets~~ balls a snake could eat before he could no longer fit in the playing field? That is, how long is a perfect game of snake?

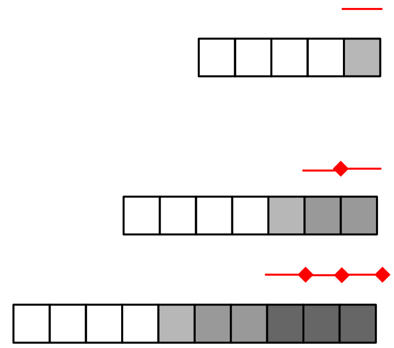
Largest number of balls a snake could eat to fit on the 32 x 32 grid.

"Snake" is a game. A player moves a snake on a square grid. The grid is 32 by 32 units.

The snake eats white balls.

The snake starts as a 4 by 1 rectangle. 

The snake grows when it eats the balls:



- The snake eats the 1st ball. Its body grows 1 square unit.
  The length grows by 1 unit. (+1 unit)
- The snake eats the 2nd ball. Its body grows 2 square units.
  The length grows by 2 more units. (+2 units)
- The snake eats the 3rd ball. Its body grows 2 square units.
  The length grows by 3 more units. (+3 units)

When the snake eats the *n*th *ball,* the length increases by *n* units.

$L(n)$ is the length of the snake after eating n balls. For example, $L(3) = 10$.

a. How long is the snake after eating 4 balls? After 5 balls? After 6 balls?

n (balls)	4	5	6
L(n) Snake Length			

b. Write a recursive rule for the function.

$L(0)$ = ________ $L(n)$ = ________________________

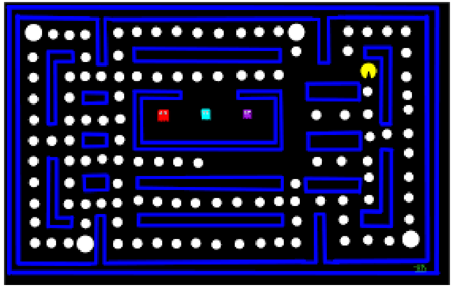
c.

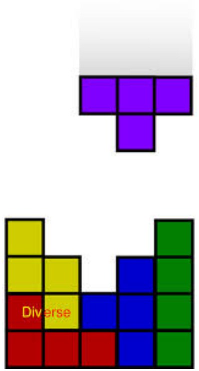
Write a non-recursive (not recursive) expression for $L(100)$. $L(100)$ = ______________________________ expression	Evaluate your expression to find $L(100)$. $L(100)$ = ________ number

d. A snake completely fills the 32x32 grid. How many balls did it eat?   n = ________

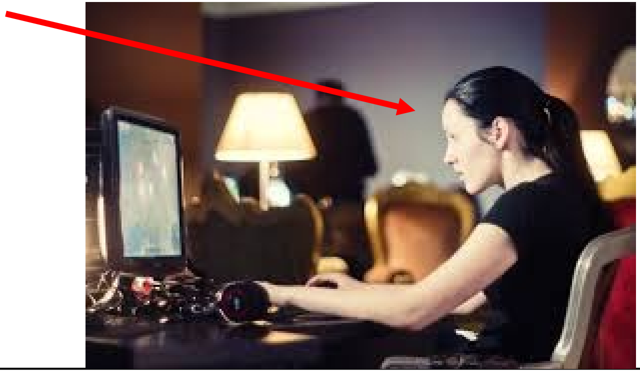
Game

EX:



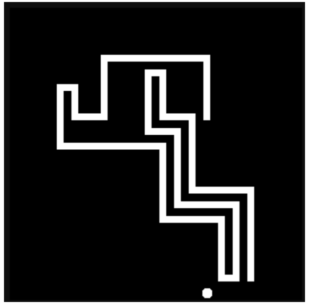


Player



Snake

Real snake

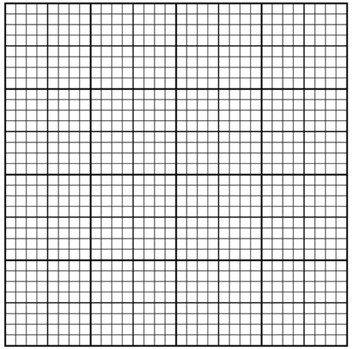


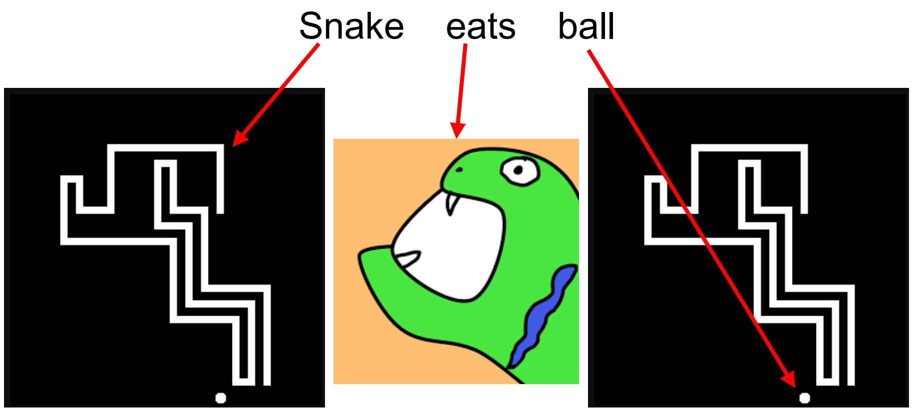
Game snake

32 by 32 grid

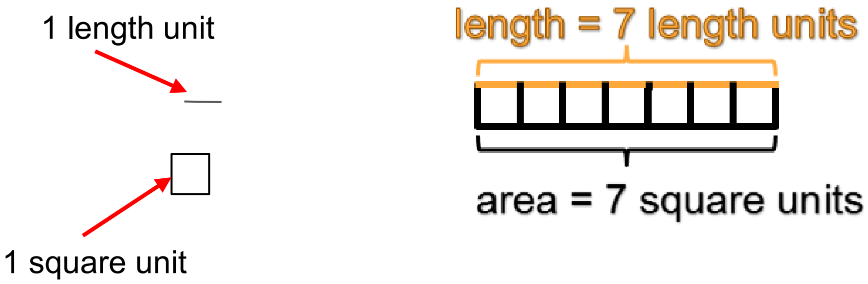
32 units

32 units

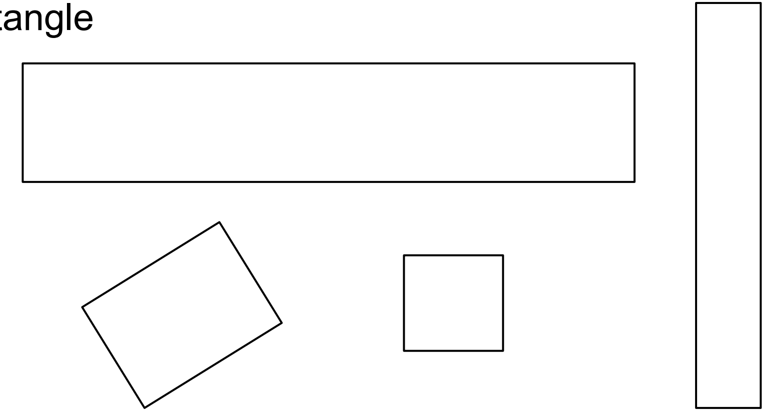




Area and Length units



Rectangle



Grow

Expression

EX:

- $4x + 2$
- $P(10) + 25$

Evaluate/Compute/Calculate

EX:

- Evaluate 4 + 2.
    - $4 + 2 = 6$
- Given $f(x) = x - 5$, evaluate $f(2)$.
    - $f(2) = 2 - 5 = -3$

Compare Explicit Form and Recursive Form