ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 6, Quadratic Relations & Conic Sections

CONNECTIONS: Michigan Academic State Standards for Mathematics

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Using the activity found at https://teacher.desmos.com/activitybuilder/custom/57c2e9a6d07333f705652027#preview/2d36fc31-5e03-4afe-a74d-22562782b259 students will listen to instructions read by the teacher as they explore different features of a parabola, use the relationships that they discover to find any vertex, sketch the associated parabolas given its focus and directrix, and justify their answers. This activity will lead students through an exploration of how moving different points affects the graph of the parabola; introduce the vocabulary of vertex, focus, and directrix; define a parabola using an applet in motion to explain the definition; and finally culminate in different problems where the student has to justify that any non-vertex point on the parabola is equidistant from the focus and directrix using different methods (e.g., counting horizontal and vertical distances on a coordinate plane, distance formula, and dynamic measurement from software) find the vertex of a parabola given the focus and directrix, and sketch the parabola given the focus and directrix. Students will explore parabolas opening up/down and right/left. The latter part of the activity introduces students to rotated parabolas and writing the equation (in conics form: y=1/4p (x-h)^2 +k) of a parabola given its vertex and focal length. Students at language proficiency levels one and two will benefit from listening to several students model appropriate language usage prior to justifying their answers. This modeling provides a necessary scaffold for students at these lower levels of language proficiency.

While the activity itself provides a box for students to submit their answers in written form, teachers could also opt to focus on speaking as students justify their answers to a partner or small group. This option is represented in the strand below.

It is important to note that the task used in the strand below has multiple pages. The supports provided are associated with specific pages of the task. The teacher is encouraged to use these sample supports with students and to create appropriate supports for the other pages in the task. For example, sentence frames are provided for pages 5 and 9; students at levels one and two will benefit from different sentence frames in order to complete other pages within the task. A teacher might also choose to modify the reference sheet provided in the supports, so that students have space to draw sketches that illuminate connections between the equations and the graphical forms, specifically what graphical features each algebraic form unveils.

COGNITIVE FUNCTION: Students at all levels of English language proficiency will **INTERPRET** oral instructions in order to examine the relationship between any point on a parabola and the focus and directrix then use this relationship to find the vertex and graph of the associated parabola given its focus and directrix and **JUSTIFY** the answer.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening and Speaking/ Writing	Interpret instructions, read aloud multiple times with purposeful pauses and pointing to an illustrated word bank, to examine the relationship between any point on a parabola and the focus and directrix, and then justify to a partner, in short phrases while pointing to the applet or sketch, how any focus and directrix can be used to find the vertex and	Interpret instructions read aloud multiple times with purposeful pauses and pointing to an illustrated word bank to examine the relationship between any point on a parabola and the focus and directrix, and then justify to a partner, in short phrases while pointing to the applet or sketch, how any focus and directrix can be used to find	Interpret instructions read aloud multiple times with purposeful pauses to examine the relationship between any point on a parabola and the focus and directrix, and then justify to a partner how any focus and directrix can be used to find the vertex and graph of the associated parabola, using an illustrated word bank, sentence stems, and a unit anchor chart.	Interpret instructions read aloud with purposeful pauses to examine the relationship between any point on a parabola and the focus and directrix, and then justify to a partner how any focus and directrix can be used to find the vertex and graph of the associated parabola, referring to a unit anchor chart.	Interpret instructions read aloud with purposeful pauses to examine the relationship between any point on a parabola and the focus and directrix, and then justify to a partner how any focus and directrix can be used to find the vertex and graph of the associated parabola, referring to a unit anchor chart.	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	graph of the associated parabola, using an illustrated word bank, sentence frames with choices, and a unit anchor chart. (Directions from page 5 of activity) "The orange [pause and point] dashed line [pause and point] is called the directrix [pause and point] of the parabola [pause and point]. Both the vertex [pause and point] and focus [pause and point], the movable [pause and point] points [pause], can move the directrix [pause and point]. Describe how [pause]." (Frame for Student Response) When you move the ______(focus, vertex)_______(up/down/right/left) the directrix moves______(up/down/left/right). (Directions from page 9 of activity) "If the dashed line [pause and point] is the directrix [pause and point] and the purple point [pause and point] is the focus [pause and point] of the parabola [pause and point], what are the coordinates [pause and point] of the vertex [pause and	the vertex and graph of the associated parabola, using an illustrated word bank, sentence frames with choices, and a unit anchor chart. (Directions from page 5 of activity) "The orange [pause and point] dashed line [pause and point] is called the directrix [pause and point] of the parabola [pause and point]. Both the vertex [pause and point] and focus [pause and point], the movable [pause and point] points [pause], can move the directrix [pause and point]. Describe how [pause]." (Frame for Student Response) When you move the ______(focus, vertex)_______(up/down/right/left) the directrix moves______(up/down/left/right). (Directions from page 9 of activity) "If the dashed line [pause and point] is the directrix [pause and point] and the purple point [pause and point] is the focus [pause and point] of the parabola	(Directions from page 5 of activity) "The orange dashed line [pause] is called the directrix [pause] of the parabola [pause]. Both the vertex [pause] and focus [pause], the movable points [pause], can move the directrix [pause]. Describe how [pause]." (Student response) When the vertex moves... When the focus moves... (Directions from page 9 of activity) "If the dashed line is the directrix [pause] and the purple point is the focus [pause] of the parabola [pause], what are the coordinates [pause] of the vertex [pause] of the parabola [pause]? How do you know? [pause]" (Student response) The coordinates of the vertex are _______. This is because...	(Directions from page 5 of activity) E.g., "The orange dashed line [pause] is called the directrix [pause] of the parabola [pause]. Both the vertex [pause] and focus [pause], the movable points [pause], can move the directrix [pause]. Describe how [pause]." (Possible Student Response) E.g., "When the vertex moves, the parameters of the equation are changing. Those changes also cause the directrix to change and move. The distance to the vertex stays the same. Moving the focus toward the vertex moves the directrix toward the vertex. Moving the focus away from the vertex moves the directrix away from the vertex." (Directions from page 9 of activity) E.g., "If the dashed line is the directrix [pause] and the purple point is the focus [pause] of the parabola [pause], what are the coordinates [pause] of the vertex [pause] of the parabola [pause]? How do you know? [pause]"	(Directions from page 5 of activity) "The orange dashed line [pause] is called the directrix [pause] of the parabola [pause]. Both the vertex [pause] and focus [pause], the movable points [pause], can move the directrix [pause]. Describe how [pause]." (Possible Student Response) E.g., "When the vertex moves, the parameters of the equation are changing. Those changes also cause the directrix to change and move. The distance to the vertex stays the same. Moving the focus toward the vertex moves the directrix toward the vertex. Moving the focus away from the vertex moves the directrix away from the vertex." (Directions from page 9 of activity) "If the dashed line is the directrix [pause] and the purple point is the focus [pause] of the parabola [pause], what are the coordinates [pause] of the vertex [pause] of the parabola [pause]? How do you know? [pause]"	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	point to vertex on the anchor chart] and point of the parabola? How do you know? [pause]" (Frame for Student Response and example response) The coordinates of the vertex are _______. [Then pointing to the vertex on the applet or sketch] "half-way" or "same distance"	[pause and point], what are the coordinates [pause and point] of the vertex [pause and point to vertex on the anchor chart] of the parabola? How do you know? [pause]" (Frame for Student Response and example response) The coordinates of the vertex are _______. [Then pointing to the vertex on the applet or sketch] "half-way" or "same distance"		(Possible Student Response) E.g. "Any point on the parabola must be the same distance from both the focus and the directrix. Since the vertex is a point on the parabola and the distance between this focus and directrix is 4 units then the vertex must be half way between. The coordinates of the vertex would be (4,1)."	(Possible Student Response) E.g. "Any point on the parabola must be the same distance from both the focus and the directrix. Since the vertex is a point on the parabola and the distance between this focus and directrix is 4 units then the vertex must be half-way between. The coordinates of the vertex would be (4,1)."	

ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 6, Quadratic Relations & Conic Sections

EXAMPLE CONTEXT FOR LANGUAGE USAGE: The standards ask students to rewrite equations of circles by completing the square to find the center and radius of the circle. The strand below illustrates how students can use the same process of completing the square to reveal key features of ellipses and hyperbolas. After completing the square either independently or with a partner, they use the new form of the equation to then describe the key features of the graph. Sketching a graph of the ellipse and hyperbola provides an additional visual support for students in making sense of the mathematics and related language.

The anchor chart and reference sheet are provided to all students to increase the opportunities for critical thinking and focus students' attention on revealing key features of graphs from algebraic forms rather than memorization of the equations for each conic section. Because of the abstract nature of these complex equations, the reference sheet allows students to analyze given equations. Analysis includes attending to the structure of the equations. Students can use this structure to identify the conic section and subsequently conjecture about how changes in the parameters of the equations affect the graphical attributes of the conics. In the strand below, the example student responses show how students use the structure in the equations to justify their thinking. In addition, as noted above, teachers may also choose to modify the reference sheet to include space where students can make sketches and illuminate connections between algebraic and graphical forms. Additional mathematical support to focus students' efforts on connections between algebraic parameters and graphical attributes in this task could include a written example for the process of completing the square with a previously studied conic section (parabolas or circles). A corresponding sketch with attributes highlighted and labeled helps illustrate language, mathematical ideas, and possible success criteria for applying this process to new conic sections (ellipses and hyperbolas).

COGNITIVE FUNCTION: Students at all levels of English language proficiency **DESCRIBE** the key features of an ellipse or hyperbola after completing the square to transform the equation.

	Level 1 **Entering**	**Level 2** **Emerging**	**Level 3** **Developing**	**Level 4** **Expanding**	**Level 5** **Bridging**	**Level 6** **Reaching**
Speaking	Describe, in words and short phrases, while pointing to a sketch, the key features of an ellipse or hyperbola after transforming the equation by completing the square, using a unit anchor chart and reference sheet. E.g., [Pointing to attributes on a sketch of an ellipse] "Horizontal major axis" [pointing to the major axis on the sketch] "Because" [Point at 36 and x in the equation] "Vertices (0, -4) and (12, -4)."	Describe in simple sentences the key features of an ellipse or hyperbola after completing the square to transform the equation using a unit anchor chart, reference sheet and sentence frames with some choices My equation is_____________. This is a(n) _____________ (ellipse/hyperbola) with a center at ____________. [Repeat each of the following as needed for an ellipse.]	Describe using complete sentences the key features of an ellipse or hyperbola after completing the square to transform the equation using a suggested word list (e.g., center, ellipse/hyperbola, major axis, minor axis, vertices/co-vertices, length, horizontal, vertical, asymptotes, transverse axis, conjugate axis) and a unit anchor chart and reference sheet. E.g., "I know 4x^2+9y^2-48x+72y+144 = 0 is an ellipse because 4x^2 and 9y^2 have the same sign. I completed the	Describe using compound and/or complex sentences the key features of an ellipse or hyperbola after completing the square to transform the equation using a suggested word list (e.g., center, ellipse/hyperbola, major axis, minor axis, vertices/co-vertices, length, horizontal, vertical, asymptotes, transverse axis, conjugate axis) and a unit anchor chart and reference sheet. E.g., "I know the equation represents an ellipse	Describe using compound and/or complex sentences the key features of an ellipse or hyperbola after completing the square to transform the equation using a suggested word list (e.g., center, ellipse/hyperbola, major axis, minor axis, vertices/co-vertices, length, horizontal, vertical, asymptotes, transverse axis, conjugate axis) and a unit anchor chart and reference sheet. E.g., "I know the equation represents an ellipse	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued		The ______(minor/major) axis is _______________ (horizontal/vertical), because ___(#) is larger than ____ (#). The equation is ________. The length is_____. The ______(co-vertices/vertices) are _____________. [Repeat each of the following as needed for a hyperbola.] The ______(conjugate/transverse) axis is _______________ (horizontal/vertical). The equation is ________. The vertices are _____________. The equations of the asymptotes are ________.	square to get ((x-6)^2)/36+((y+4)^2)/16=1. This is an ellipse with a horizontal major axis, because 36 > 16. It has a center at (6, -4). It has a horizontal major axis at y = - 4. The length is 12. The vertices are (0, -4) and (12, -4). It has a vertical minor axis at x = 6. The length is 8. The co-vertices are (6, 0) and (6, -8). There are no asymptotes."	because A and C have the same sign (4 and 9). I completed the square to rewrite 4x^2+9y^2-48x+72y+144 = 0 as ((x-6)^2)/36+((y+4)^2)/16=1. I know that this is an ellipse with a center at (6, -4). It has a horizontal major axis, because 6 > 4 (36 > 16). The major axis is at y = - 4 with a length of 12 and vertices at (0, -4) and (12, -4). It also has a vertical minor axis at x = 6, a length of 8, and co-vertices at (6, 0) and (6, -8). There are no asymptotes."	because A and C have the same sign (4 and 9). I completed the square to rewrite 4x^2+9y^2-48x+72y+144 = 0 as ((x-6)^2)/36+((y+4)^2)/16=1. I know that this is an ellipse with a center at (6, -4). It has a horizontal major axis, because 6 > 4 (36 > 16). The major axis is at y = - 4 with a length of 12 and vertices at (0, -4) and (12, -4). It also has a vertical minor axis at x = 6, a length of 8, and co-vertices at (6, 0) and (6, -8). There are no asymptotes."	

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: In this task students explore transformations of conic sections, many of which are not written in function form. Thus, this task is an extension of the following standard:
HSF-BF.B.3: *Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Students are given equations of conic sections and experiment with changing the parameters within these equations. Graphing technology (e.g., Desmos, graphing calculators) scaffolds the inquiry and serves as a tool for students to verify or modify their conjectures. In addition, graphical representations serve as a language scaffold. In the strand below, students at levels one and two may use a combination of sentence frames and labeled sketches as different ways to describe the results of changing the parameters with the language of transformations. For students at levels one and two, teachers are encouraged to accept both labeling and sentence writing as acceptable evidence of understanding.

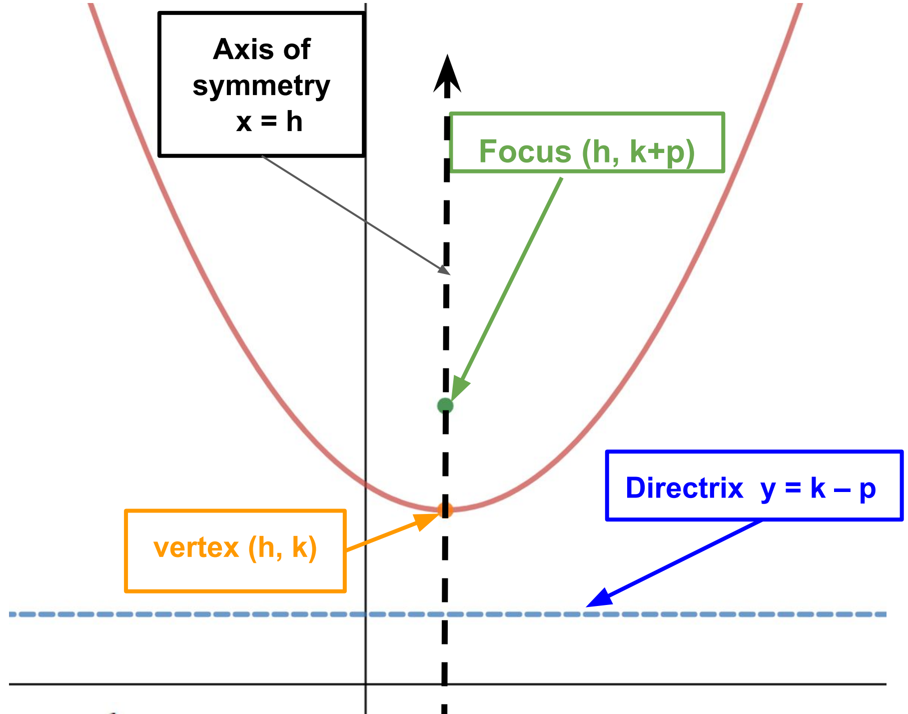
It is important to note that the different conic sections have different key features that would need to be described. The teacher is encouraged to use these sample supports with students and to create appropriate supports for the other pages in the task. For example, generic sentence frames are provided for this type of task.

COGNITIVE FUNCTION: Students at all levels of English language proficiency identify and **EXPLAIN** the relationships between changes in the equations and changes in the graphs of quadratic relations and conic sections.

	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Writing	Explain in simple sentences, words, and/or phrases how changes in the equation of a quadratic relation or conic section affect the key features of its graph while referring to a transformation anchor chart, a unit anchor chart and reference sheet, while working with a partner, when the task is broken into smaller components and sentence frames are provided. Part 1: What is your original equation? What conic section does it represent? My original equation is ___________. It represents a(n) _________ (ellipse,	Explain in simple sentences and/or phrases how changes in the equation of a quadratic relation or conic section affect the key features of its graph while referring to a transformation anchor chart, a unit anchor chart and reference sheet, while working with a partner, when the task is broken into smaller components and sentence frames with choices are provided. Part 1: What is your original equation? What conic section does it represent?	Explain in complete sentences how changes in the equation of a quadratic relation or conic section affect the key features of its graph while referring to a transformation anchor chart, a unit anchor chart and reference sheet, while working with a partner, when the task is broken into smaller components. Part 1: What is your original equation? What conic section does it represent? Identify the key features (center, vertices/co-vertices, major/minor axes, focus, directrix, transverse/conjugate axes).	Explain in complex and/or compound sentences how changes in the equation of a quadratic relation or conic section affect the key features of its graph while referring to a unit anchor chart and reference sheet, while working with a partner. E.g., "My original equation is (x^2)/9 + (y^2)/16 = 1, which is an ellipse with a center at (0, 0), a vertical major axis of length 8, and a horizontal minor axis of length 6. The vertices are at (0, 4), (0, -4), (3, 0) and (-3, 0). My transformed	Explain in complex and/or compound sentences how changes in the equation of a quadratic relation or conic section affect the key features of its graph while referring to a unit anchor chart and reference sheet and working with a partner. E.g., "My original equation is (x^2)/9 + (y^2)/16 = 1, which is an ellipse with a center at (0, 0), a vertical major axis of length 8, and a horizontal minor axis of length 6. The vertices are (0, 4), (0, -4), and co-vertices are (3, 0) and (-3,	

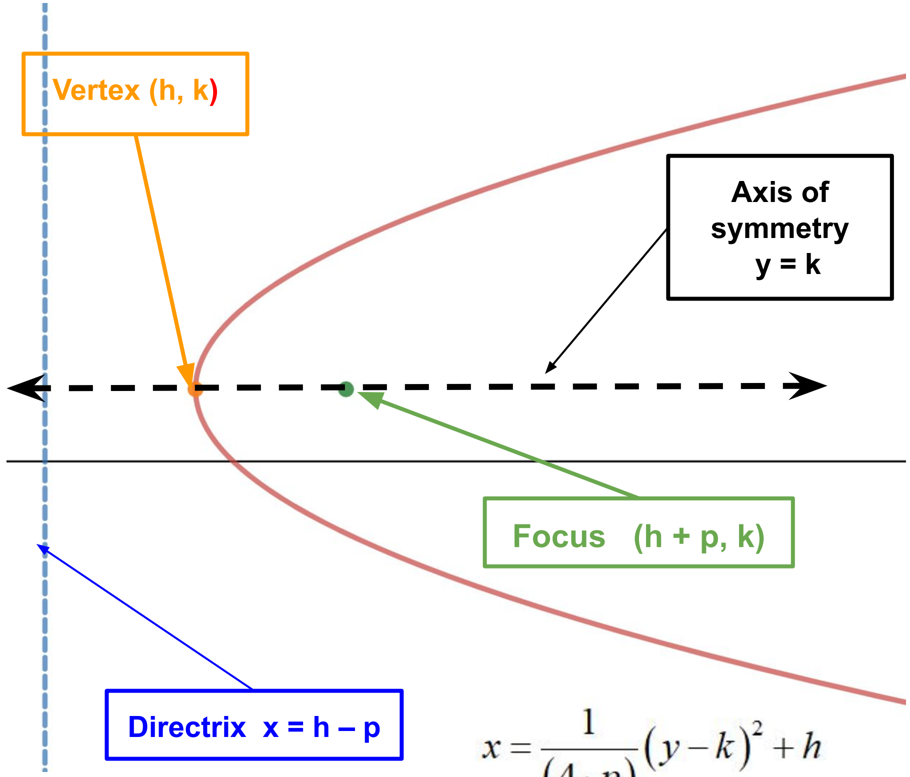
	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	hyperbola, parabola). Part 2: Draw the graph. Label the key features. [Student labels features on a sketch.] Part 3: What is your transformed equation? My transformed equation is ________. Draw the graph. Label the key features. [Student labels features on a sketch.] Part 4. How did the equation transform the graph? This is a _______________ (translation, stretch/shrink, rotation, reflection) ____________ (how/in what direction) ____________ (how much) (Repeat as needed.)	My original equation is ______. It represents a(n) _______ (ellipse, hyperbola, parabola). Part 2: Identify the key features. [Use frames as needed] The ______(center/vertex/ focus) is (___,___) The equation of the directrix is_________. The vertices/co-vertices are _________ (list). The major axis is _____ (#) units long. The minor axis is _____ (#) units long. Part 3: What is your transformed equation? Identify the key features of the graph. My transformed equation is ________. The ______(center/vertex/ focus) is (___,___) The equation of the directrix is_________. The vertices/co-vertices are _________ (list). The major axis is _____ (#) units long. The minor axis is _____ (#) units long. Part 4. How did the equation transform the graph? This is a __________ (translation, dilation, rotation, reflection) _______ (how much?) __________ (in what direction?) (Repeat as needed.)	E.g., "My original equation is (x^2)/9 + (y^2)/16 = 1. It is an ellipse. The center is (0, 0). The vertices are (0, 4), (0, -4). The co-vertices are (3, 0) and (-3, 0). The major axis is vertical, 8 units long, and the minor axis is horizontal, 6 units long." Part 2: What is your transformed equation? How did it affect the key features of the graph? E.g., "My transformed equation is [(x-2)^2]/9 + [(y+5)^2]/16 = 1. This is a translation 2 units to the right and 5 units down. So the center is at (2, -5). The size of the ellipse is the same. The major axis is still 8 units long and the minor axis is 6 units long. The vertices are (2,-1) and (2,-9). The co-vertices are (5,-5) and (-1,-5)."	equation is [(x-2)^2]/9 + [(y+5)^2]/16 = 1, which has the same dimensions, but is translated 2 units to the right and 5 units down from the origin. The new center is (2,-5). The vertices are (2,-1) and (2,-9), and the co-vertices are (5,-5) and (-1,-5)."	0). My transformed equation is [(x-2)^2]/9 + [(y+5)^2]/16 = 1, which has the same dimensions, but is translated 2 units to the right and 5 units down from the origin. The new center is (2,-5). The vertices are (2,-1) and (2,-9), and the co-vertices are (5,-5) and (-1,-5)."	

Parabolas



$$y=\frac{1}{(4\cdot p)}(x-h)^2+k$$

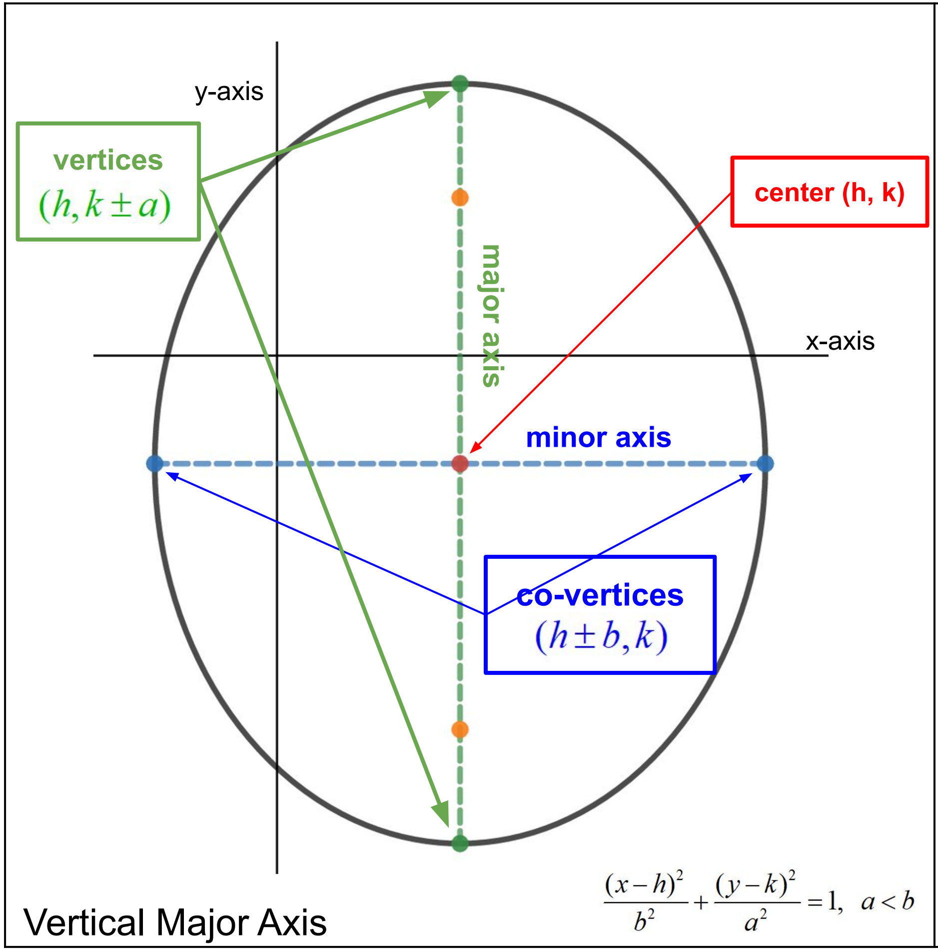
Opens up Vertical Axis of Symmetry



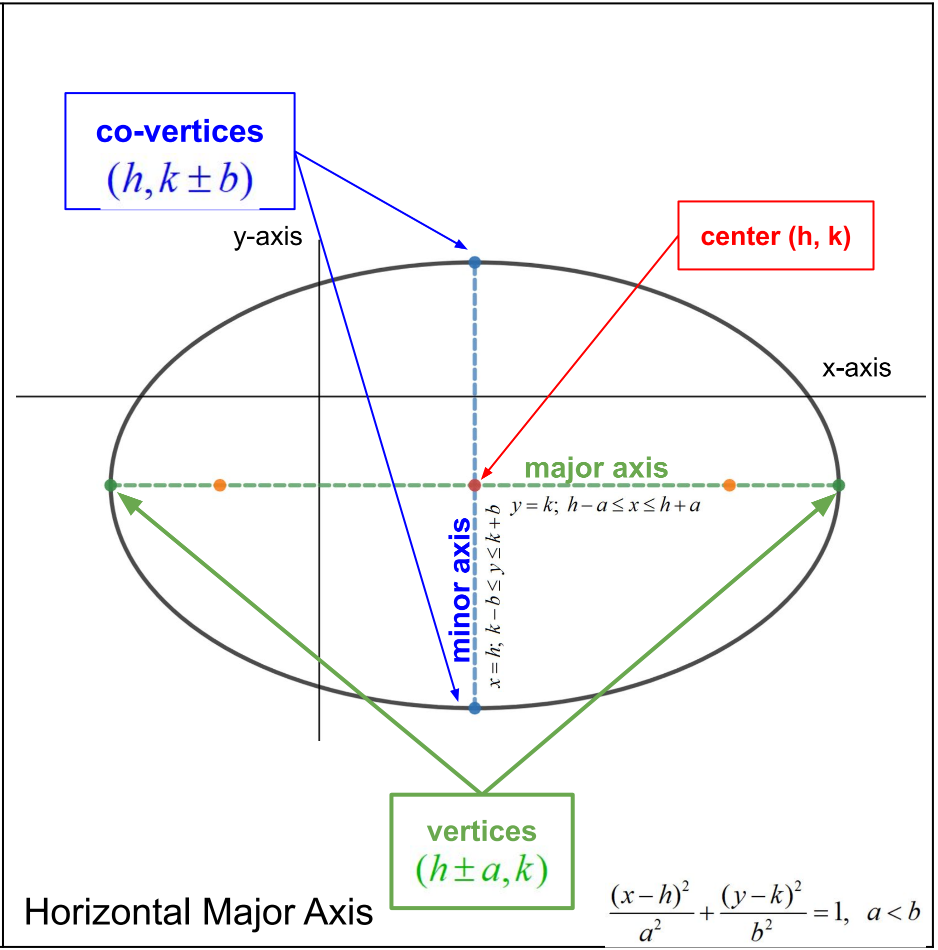
$$x=\frac{1}{(4\cdot p)}(y-k)^2+h$$

Opens right Horizontal Axis of Symmetry

Ellipses

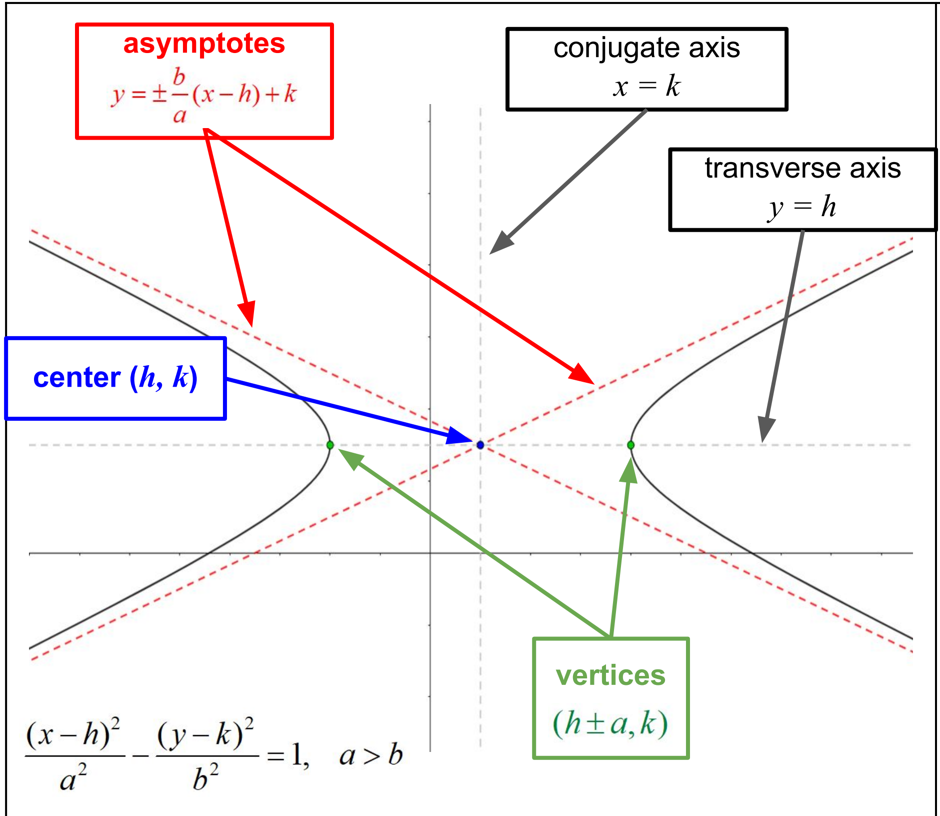


y-axis
vertices
$(h, k \pm a)$
center (h, k)
major axis
x-axis
minor axis
co-vertices
$(h \pm b, k)$
Vertical Major Axis
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a < b$

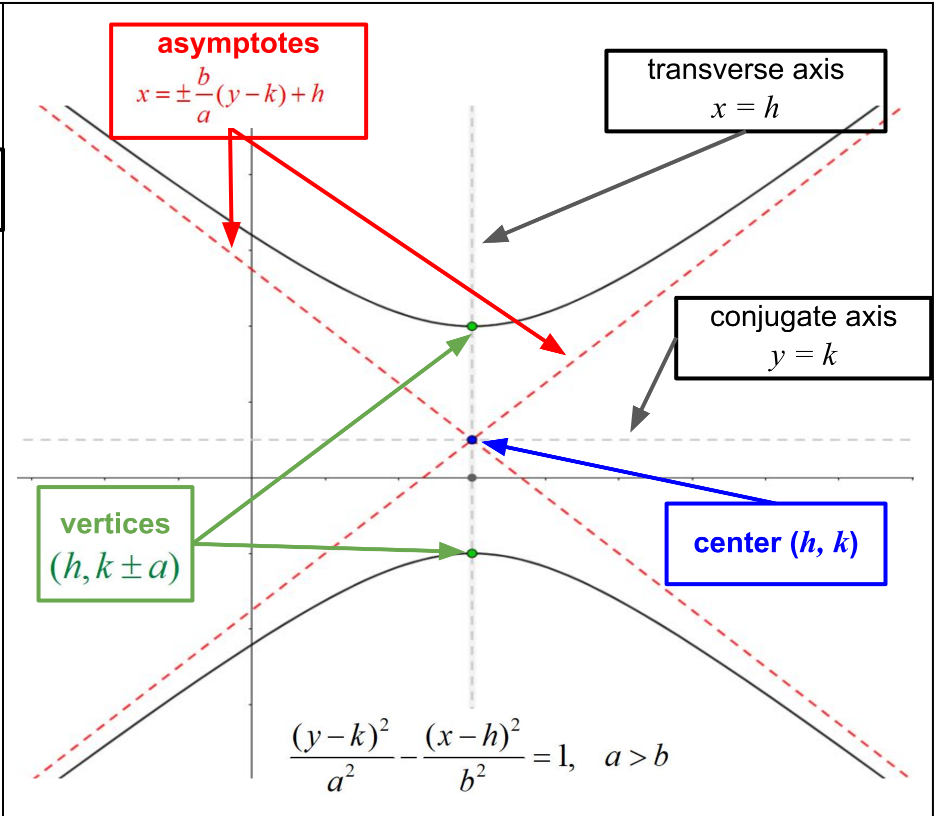


co-vertices
$(h, k \pm b)$
y-axis
center (h, k)
x-axis
major axis
$y = k;\ h - a \le x \le h + a$
minor axis
$x = h;\ k - b \le y \le k + b$
vertices
$(h \pm a, k)$
Horizontal Major Axis
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a < b$

Hyperbolas

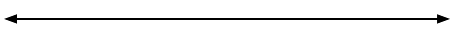


Opens left/right Horizontal Transverse Axis



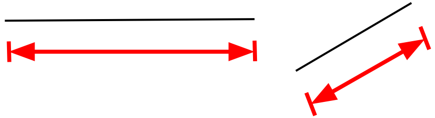
Opens up/down Vertical Transverse Axis

Horizontal



Vertical

Length



Parabola Equations

$$x = Ay^2 + By + C$$
$$= A(y-k)^2 + h$$
$$= \frac{1}{4p}(y-k)^2 + h$$

$$y = Ax^2 + Bx + C$$
$$= A(x-h)^2 + k$$
$$= \frac{1}{4p}(x-h)^2 + k$$

Ellipse Equations:

A and C have the same sign

$$Ax^2 + Bx + Cy^2 + Dy + E = 0$$
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1, \quad a > b$$

Hyperbola Equations:

A and C have opposite signs

$$Ax^2 + Bx + Cy^2 + Dy + E = 0$$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \quad a > b$$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, \quad a > b$$

Circle Equations:

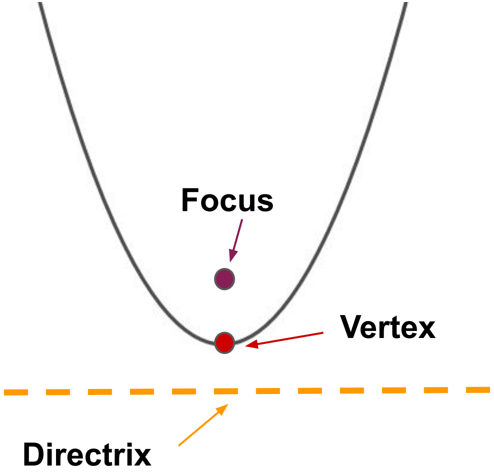
$$Ax^2 + Bx + Cy^2 + Dy + E = 0$$
$$(x-h)^2 + (y-k)^2 = r^2$$

Purple **Red** **Orange**

Dashed / Dotted

important line, but not part of graph / points that make an equation true

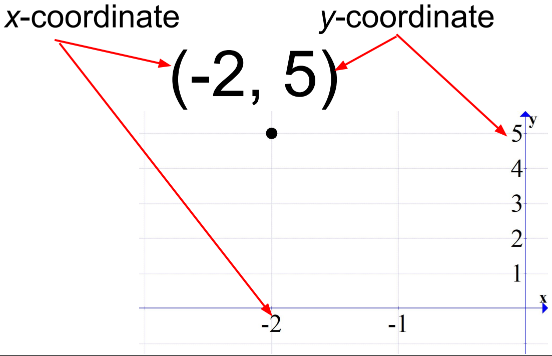
Parabola

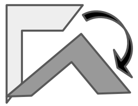
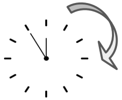
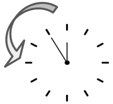
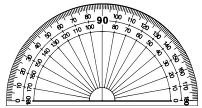
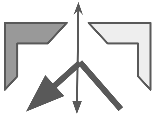
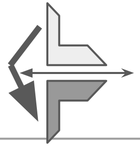
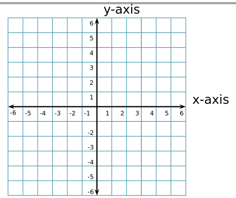
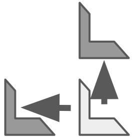
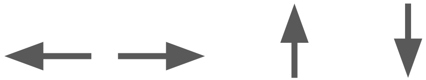
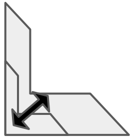
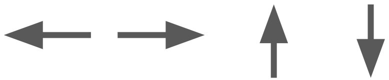


Movable

Not Movable

Coordinates



TRANSFORMATION	HOW	HOW MUCH
Rotation/Rotate 	clockwise / counterclockwise around the point (x ,y)   	______(#) degrees 
Reflection/Reflect  	over the ______(x-axis/ y-axis/ line____) 	
Translation/Translate 	shifts/moves (left / right // up / down) (horizontally // vertically) 	______(#) units
Dilation/Dilate 	horizontally / vertically 	by a factor of ______(#)