Unit 7 - Trigonometric Functions
Algebra II

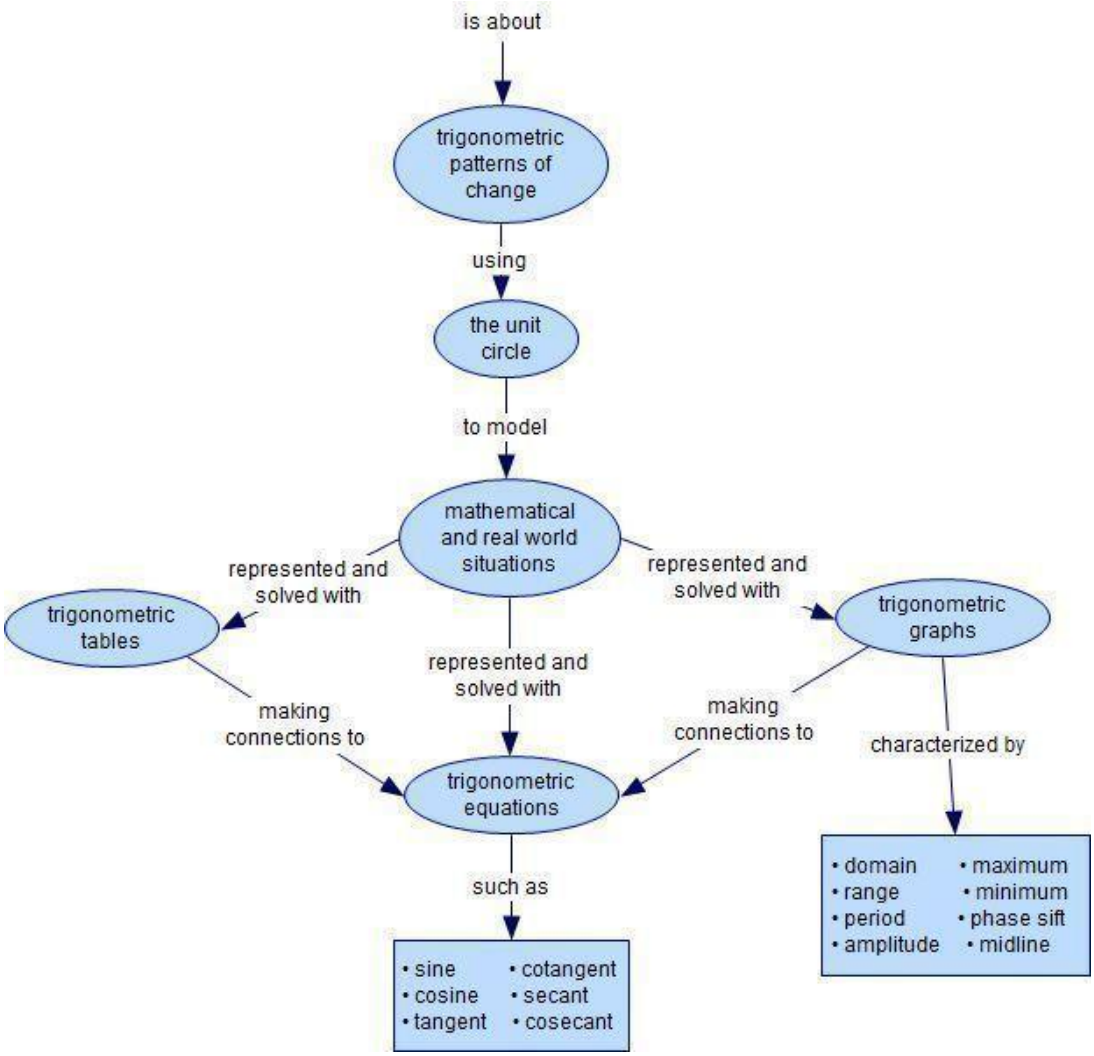
Thursday, June 16, 2016, 1:31PM

Common Core Initiative

Overarching Questions and Enduring Understandings

How can the unit circle be used to develop a circular definition of trigonometric functions?

Graphic Organizer



Unit Abstract

Previous studies of trigonometry have included right triangle trigonometric relationships and the Laws of Sines and Cosines. In this unit students develop a circular definition of trigonometric functions using both degree and radian measure. The unit circle is the basis for these definitions and leads to graphs of the sine and cosine curves. Students can then define the tangent and inverse trigonometric functions based on the sine and cosine. Making connections between the other trigonometric functions and the graphs and tables of the sine and cosine functions provides a context for students to develop the graphs and tables of the tangent, cotangent, secant, and cosecant functions.

Building on their previous knowledge of transformations (both algebraic and geometric), students explore and graph the effects of transformations (amplitude, period, and phase shift) on the sine and cosine curves.

Content Expectations/Standards

High School: Functions

Building Functions

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.4. Find inverse functions.
  - HSF-BF.B.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = (x+1)/(x–1)$ for x ? 1.
  - HSF-BF.B.4c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - HSF-BF.B.4d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Trigonometric Functions

HSF-TF.A. Extend the domain of trigonometric functions using the unit circle.

- HSF-TF.A.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- HSF-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- HSF-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4 and p/6, and use the unit circle to express the values of sine, cosines, and tangent for x, p + x, and 2p – x in terms of their values for x, where x is any real number.
- HSF-TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

HSF-TF.B. Model periodic phenomena with trigonometric functions.

- HSF-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- HSF-TF.B.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- HSF-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions
HSF-BF.A. Build a function that models a relationship between two quantities.

- HSF-BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Developed and/or utilized across multiple units

Quantity
HSN-Q.A. Reason quantitatively and use units to solve problems.

- HSN-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.
- HSN-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpreting Functions
HSF-IF.A. Understand the concept of a function and use function notation.

- functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpreting Functions
HSF-IF.B. Interpret functions that arise in applications in terms of the context.

- HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end

technology, and interpret them in terms of the context.

HSF-TF.C. Prove and apply trigonometric identities.

- HSF-TF.C.8. Prove the Pythagorean identity $\sin^2(?) + \cos^2(?) = 1$ and use it to calculate trigonometric ratios.
- HSF-TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

behavior; and periodicity. ★

- HSF-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Building Functions
HSF-BF.A. Build a function that models a relationship between two quantities.

- HSF-BF.A.1. Write a function that describes a relationship between two quantities.
  - HSF-BF.A.1b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Essential/Focus Questions

1. What is the relationship between degree and radian measures of angles? Why or when would you use degree or radian?
2. How can the effect of transformations on the sine and cosine curves be seen in the graphs and tables of these functions?
3. How can the unit circle be used to generate the sine and cosine graphs?
4. Why are the trigonometric functions periodic?
5. How do you know if a function is periodic?

Key Concepts

amplitude
asymptote
cosecant
cosine
cotangent
degree and radian relationship and conversion
domain
maxima
minima
period
phase shift
range
relate graphs of trigonometric functions to their inverses
secant
sine
tangent
transformations of trig function from the parent functions
unit circle

Assessment Tasks

- Assessment Overview
- Assessment Independent Practice Handout Option 1
- Assessment Independent Practice Handout Option 2

Intellectual Processes

Standards for Mathematical Practice

Students will have opportunities to:

- **look for and express regularity in repeated reasoning** observing the relationships between the sine and cosine and using the unit circle to generate the periodic graphs of these functions;
- **model with mathematics** using trigonometric functions and transformations of these functions to model periodic behavior; and
- **use appropriate tools strategically** such as graphing calculators or computer applets to analyze the sine and cosine functions and changes in their characteristics as parameter changes are made and to find values of trigonometric functions for specific angle measures.

Lesson Sequence

- Lesson Overview
- Lesson Handout - Changing Sines

Resources

- Unit Resources
- Unit Circle Project
- Professional Learning Task –Student Work Sample: Examining Student Work from the Unit Circle Project