ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 7, Trigonometric Functions

CONNECTIONS: Michigan State Academic State Standards

EXAMPLE CONTEXT FOR LANGUAGE USAGE: Students are playing a two player game. Each player secretly chooses one graph from an assortment of trigonometric functions and leaves it in the collection for his partner to see. Then the students take turns asking questions about the attributes of their partners' functions (e.g., amplitude, period, maximum/minimum, intercepts, end behavior, symmetry) one at a time. With each question, partners narrow down the possible choices and eventually determine which graph was chosen by his partner. This strand is based on the Desmos activity at https://teacher.desmos.com/polygraph/custom/5592c482b1d9824f46eda37b exploring sine and cosine curves. When participating in the activity online, students write questions that their partner reads and answers by clicking yes or no. The activity is modified below, so that student partners are sitting next to each other, asking and answering each other's questions verbally. This offline modification also allows the flexibility for students to ask questions requiring more than a yes/no response (e.g., "On what interval is your function increasing?").

There are two strands below that address this activity, one for listening and the other for speaking. Note for the listening strand, students only need to respond with yes/no answers. In the speaking strand, students may use pointing and gesturing to clarify their questions or enhance their pronunciation. Pointing and gesturing was also used in the listening strand as a simultaneous scaffold for the listener.

HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

LISTENING COGNITIVE FUNCTION: Students at all levels of English language proficiency will **SYNTHESIZE** questions about trigonometric functions asked by a classmate.

SPEAKING COGNITIVE FUNCTION: Students at all levels of English language proficiency will **DESCRIBE** key features of a trigonometric function based on questions from a classmate.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Listening	Synthesize questions about trigonometric functions asked multiple times while the speaker is pointing to copy of the unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Synthesize questions about trigonometric functions asked multiple times while the speaker is pointing to copy of the unit anchor chart E.g., “Does the function have a y-intercept at (0, 0)?”, Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Synthesize questions about trigonometric functions asked multiple times using a unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, "Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Synthesize questions about trigonometric functions using a unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Synthesize questions about trigonometric functions using a unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, “Is the function increasing between x = 0 and x = pi/2?” “Is the amplitude 2?”	

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	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Speaking	Ask simple questions to a partner about the key features of a trigonometric functions using a suggested word list (e.g., y-intercept, x-intercepts, positive, negative, increasing, decreasing, interval, period, amplitude, maximum, minimum, symmetric), sentence stems/frames with choices, pointing to a personal copy of the unit anchor chart or gesturing about key features. Is/are the ______(amplitude, period, y-intercept, x-intercepts, maximum, minimum)...? Is the function ________(increasing/decreasing) [point to interval]?	Ask questions in simple sentences to a partner about the key features of a trigonometric functions using a suggested word list (e.g., y-intercept, x-intercepts, positive, negative, increasing, decreasing, interval, period, amplitude, maximum, minimum, symmetric), question stems, pointing to a personal copy of the unit anchor chart or gesturing about key features. Does the function have...? Is the function...? Is/are the ______(amplitude, period, y-intercept, x-intercepts, maximum, minimum, intercepts)...?	Ask questions in simple sentences about the key features of trigonometric functions using a suggested word list (e.g., y-intercept, x-intercepts, positive, negative, increasing, decreasing, interval, period, amplitude, maximum, minimum, symmetric) and the unit anchor chart. E.g., “Is the y-intercept (0, 0)?”, "Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Ask questions in complete sentences about the key features of trigonometric functions using a suggested word list (e.g., y-intercept, x-intercepts, positive, negative, increasing, decreasing, interval, period, amplitude, maximum, minimum, symmetric) and the unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, "Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	Ask questions in complete sentences about the key features of trigonometric functions using a suggested word list (e.g., y-intercept, x-intercepts, positive, negative, increasing, decreasing, interval period, amplitude, maximum, minimum, symmetric) and the unit anchor chart. E.g., “Does the function have a y-intercept at (0, 0)?”, "Is the function increasing between x = 0 and x = pi/2?”, “Is the amplitude 2?”	

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: Periodic functions in real life contexts provide rich opportunities for students to apply and make sense of trigonometric functions. The task below from https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/595, is a culminating activity where students draw from multiple mathematical content and experiences (e.g, circumference of a circle and the relationship between distance, rate, and time.) All students would benefit from the scaffold of working in small groups. While the language in the problem may not seem sophisticated, the complexity of the mathematics necessitates the use of language supports. For this task, the unit anchor chart provides a scaffold for *mathematical* reasoning. The use of graphing utilities also scaffolds mathematical reasoning and opens space for students to *analyze* the graph (rather than produce) and make connections to the context. Although the simplified student copy of the task is designed to support students at levels 1 and 2, other students may benefit from using the support to help make meaning of the more complicated text.

The illustrated word list in the supports for this strand is in addition to the other supports named in the strand below. This support may be used for pre-teaching academic vocabulary related to the complex mathematical text. The mathematics teacher, ESL teacher, or a paraprofessional, may find a few minutes before the start of this lesson to use this resource with students needing explicit instruction of these vocabulary words. Alternatively, this support could be used in place of the simplified illustrated version support, as appropriate based on student needs.

The task addresses the standard HSF-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** a linguistically complex mathematical text.

	Level 1 Entering	**Level 2 Emerging**	**Level 3 Developing**	**Level 4 Expanding**	**Level 5 Bridging**	**Level 6 Reaching**
Reading	Analyze a simplified, illustrated, linguistically complex mathematical text in order to model a periodic phenomenon, while referring to a unit anchor chart and working in a small group. A simplified, illustrated student copy is found in the supports for this unit.	Analyze a simplified, illustrated, linguistically complex mathematical text in order to model a periodic phenomenon, while referring to a unit anchor chart and working in a small group. A simplified, illustrated student copy is found in the supports for this unit.	Analyze a glossed linguistically complex mathematical text in order to model a periodic phenomenon, while referring to a unit anchor chart and working in a small group. A glossed student copy is found in the supports for this unit.	Analyze a linguistically complex mathematical text in order to model a periodic phenomenon, while referring to a unit anchor chart and working in a small group. A student copy is found in the supports for this unit.	Analyze a linguistically complex mathematical text in order to model a periodic phenomenon, while referring to a unit anchor chart and working in a small group. A student copy is found in the supports for this unit.	

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: The strand below addresses the mathematics in the second part of the standard, HSF-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and **use the unit circle to express the values of sine, cosines, and tangent for x, pi + x, and 2pi – x in terms of their values for x**, where x is any real number. It is assumed that students have already determined the relationship between sine, cosine, and tangent and are applying that relationship to determine values of tangent.

In this strand, students explain how to use the unit circle to express the values of sine, cosine, and tangent for any multiple of pi/3, pi/4 and pi/6. Students at all levels of proficiency are encouraged to work in pairs and small groups to make sense of the mathematics and use language informally prior to engaging in the writing task which requires the use of precise mathematical language. One example is shown below.

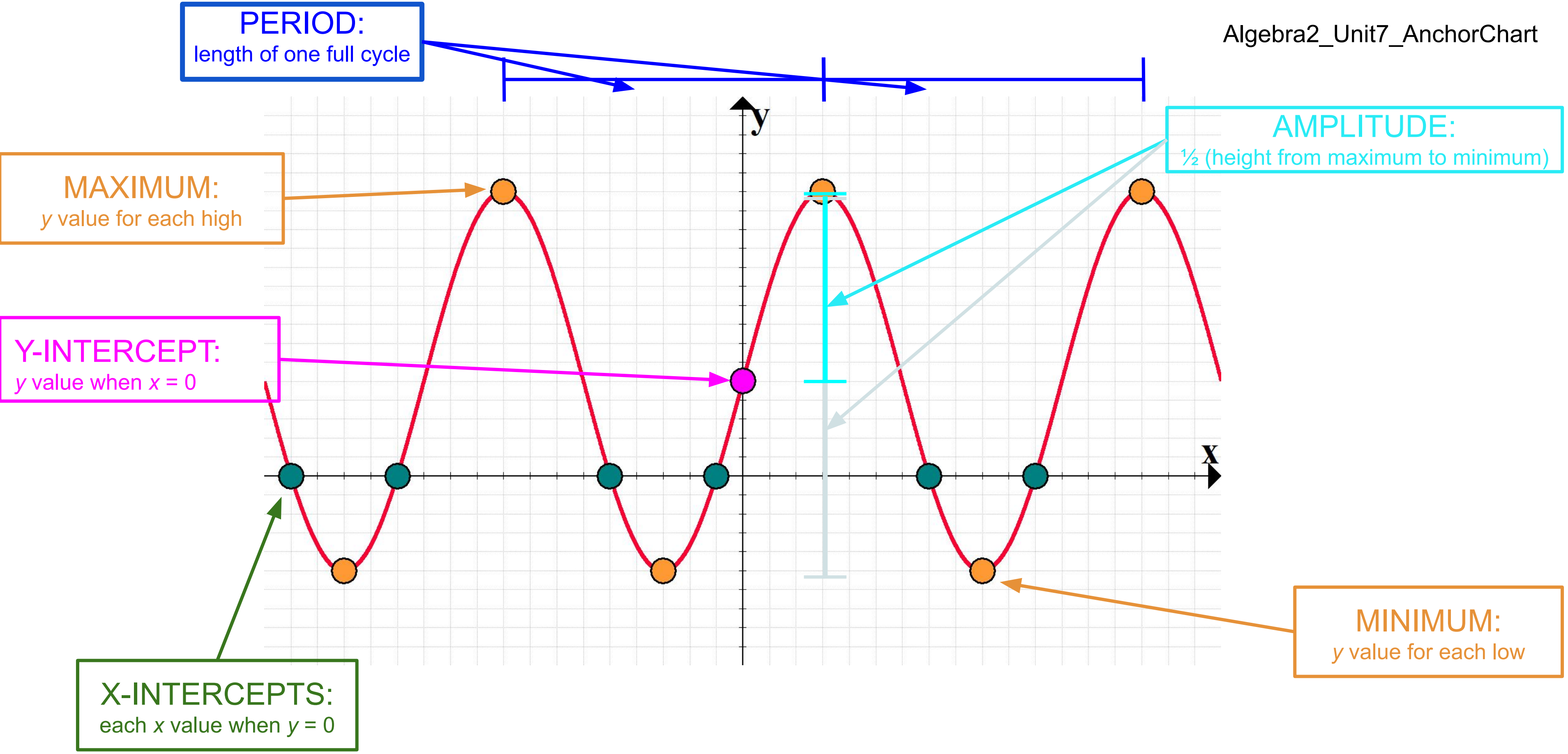
Students create mathematical meaning as they produce language. In order to scaffold mathematical reasoning a teacher might also ask questions like: What is the closest whole number multiple of pi? (e.g., “for 23*pi/4, the closest whole number of pi will be 6*pi = 24*pi/4”. Note, when there are two whole number values, like with 3pi/2, students choose one of the two.) How many times around the unit circle is that? What is the reference angle? What is the quadrant? As with any scaffolding strategies, teachers should assess students' readiness. Providing scaffolds too early has potential to diminsh opportunities for students make sense of the problem and design their own path for solving and communicating. Similarly, when creating sentence frames and stems to support academic language, teachers need to be mindful of overscaffolding in order to maintain the cognitive demand of the mathematics.

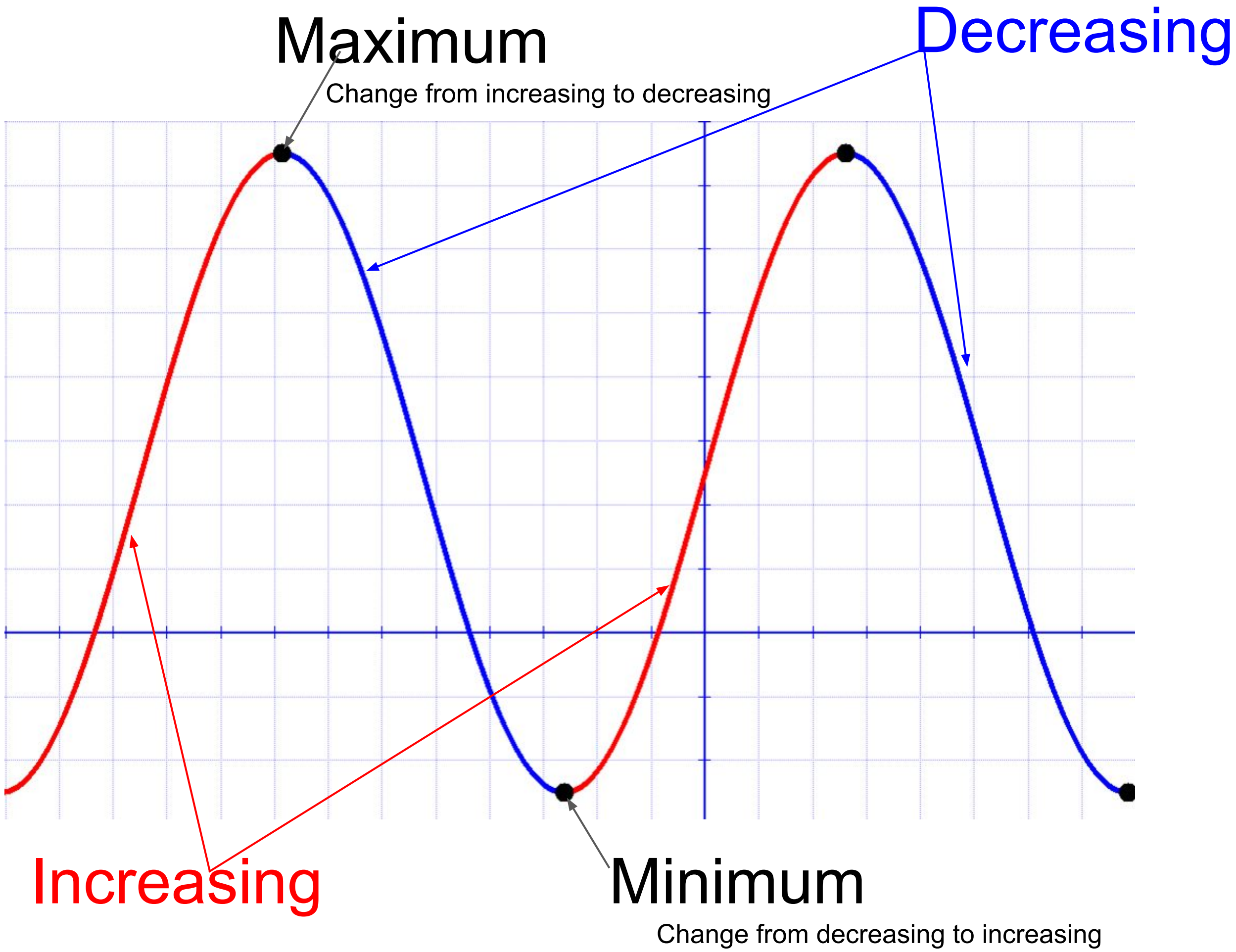
Using a laminated unit circle and dry erase marker allows students to rehearse and receive feedback on the precision of their labeling as a form of explanation.

COGNITIVE FUNCTION: Students at all levels of English language proficiency **EXPLAIN** how to use the unit circle to find the values of sine, cosine, and tangent for any multiple of pi/3, pi/4 and pi/6.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Writing	Explain in simple sentences, phrases, and/or labeling how to use the unit circle to find the exact value of sine, cosine, or tangent ratios given an angle measured in radians, using a unit circle anchor chart, an angle reference sheet and working with a partner, when the task is broken into smaller components and sentence frames are provided. Part 1: Describe or show how to find the reference angle and quadrant for your angle. [The student writes sentences using the frames	Explain in simple sentences, phrases, and/or labeling how to use the unit circle to find the exact value of sine, cosine, or tangent ratios given an angle measured in radians, using a unit circle anchor chart, an angle reference sheet and working with a partner, when the task is broken into smaller components and sentence frames are provided. Part 1: Describe or show how to find the reference angle and quadrant for your angle. [The student writes	Explain in complete sentences how to use the unit circle to find the exact value of sine, cosine, or tangent ratios given an angle measured in radians, using a unit circle anchor chart, an angle reference sheet, a suggested word list (i.e. unit circle, reference angle, positive/negative, sine/cosine/tangent, quadrant) and working with a partner, when the task is broken into smaller components. Part 1: Describe how to find the reference angle and quadrant for your angle. E.g., "18pi/6 = 3pi which is one and a half times around	Explain in compound and/or complex sentences how to use the unit circle to find the exact value of sine, cosine, or tangent ratios given an angle measured in radians, using an anchor chart, a suggested word list (i.e. unit circle, reference angle, positive/negative, sine/cosine/tangent, quadrant) and working with a partner. E.g., "Since 18pi/6=3pi and would be one and a half times around the unit circle, 19pi/6 is a little more than that and would be in Quadrant III giving a reference angle of pi/6.	Explain in compound and/or complex sentences how to use the unit circle to find the exact value of sine, cosine, or tangent ratios given an angle measured in radians using a unit circle anchor chart, a required word list (i.e. unit circle, reference angle, positive/negative, sine/cosine/tangent, quadrant) and working with a partner. E.g., "Since 18pi/6=3pi and would be one and a half times around the unit circle, 19pi/6 is a little more than that and would be in Quadrant III giving a reference angle of pi/6.	

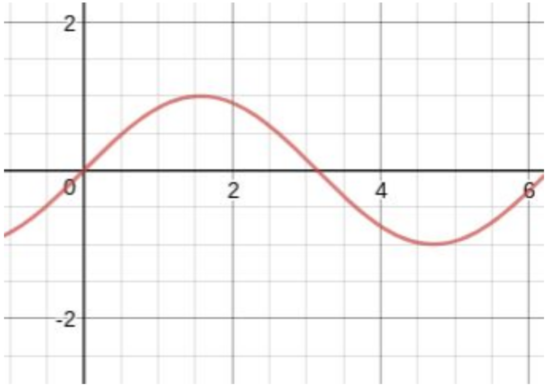
	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued	below and labels a diagram illustrating how the given angle relates to the unit circle.] The angle is in quadrant ______(number). The reference angle is _______. Part 2: What is the value of sine/cosine/tangent for the reference angle in Quadrant I? Does that value change in the quadrant you found in part 1? The reference angle______ has a _______(sine/cosine/tangent) value of ____ in Quadrant I. In Quadrant _______(from part 1), the value is ____(+/-) _____(#). Part 3: The exact value is_____.	sentences using the frames below and labels a diagram illustrating how the given angle relates to the unit circle.] The angle is in quadrant ______(number). The reference angle is _______. Part 2: What is the value of sine/cosine/tangent for the reference angle in Quadrant I? Does that value change in the quadrant you found in part 1? The reference angle______ has a _______(sine/cosine/tangent) value of ____ in Quadrant I. In Quadrant _______(from part 1), the value is ____(+/-)_____(#). Part 3: The exact value is_____.	the unit circle. 19pi/6 is a little more than that. It would be in Quadrant III. The reference angle is pi/6." Part 2: What is the sine/cosine/tangent value in Quadrant I for the reference angle? Does that value change in the quadrant you found in part 1? E.g., "In Quadrant I, pi/6 has a sine value of 1/2. In Quadrant III, the sine value is negative. Part 3: What is the exact value? E.g., "The exact value of sin(19pi/6) is -1/2."	Pi/6 is in the first quadrant and has a sine value of 1/2, so 19pi/6 will have a sine value of -1/2 because in Quadrant III all sine values are negative."	Pi/6 is in the first quadrant and has a sine value of 1/2, so 19pi/6 will have a sine value of -1/2 because in Quadrant III all sine values are negative."	


PERIOD:
length of one full cycle
AMPLITUDE:
½ (height from maximum to minimum)
MAXIMUM:
y value for each high
Y-INTERCEPT:
y value when x = 0
MINIMUM:
y value for each low
X-INTERCEPTS:
each x value when y = 0
y
x

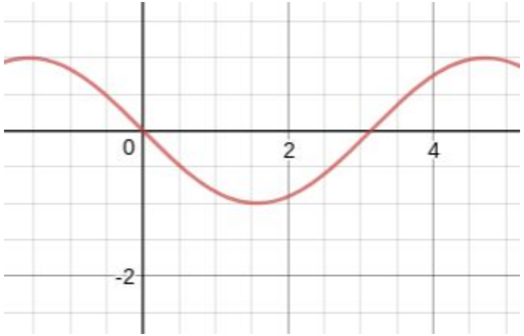

Maximum
Change from increasing to decreasing
Decreasing
Increasing
Minimum
Change from decreasing to increasing

Sine Functions

$f(x) = sin\Theta$

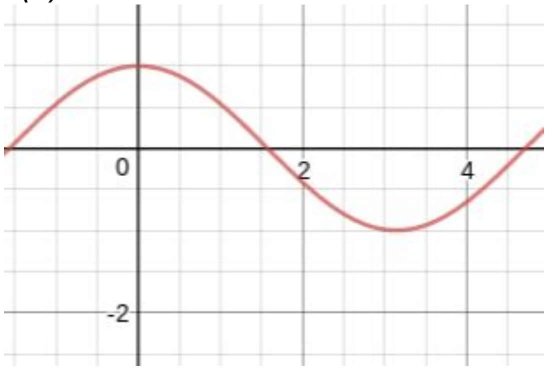


$f(x) = -sin\Theta$

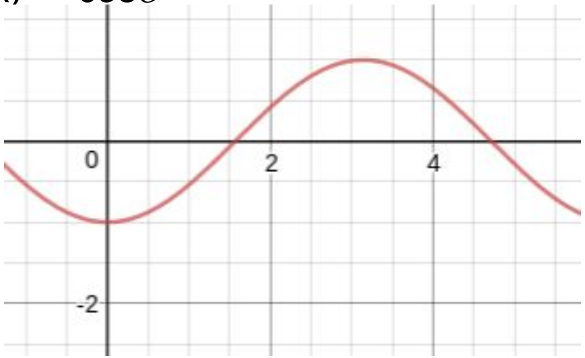


Cosine Functions

$f(x) = cos\Theta$



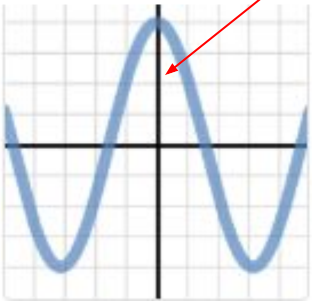
$f(x) = -cos\Theta$

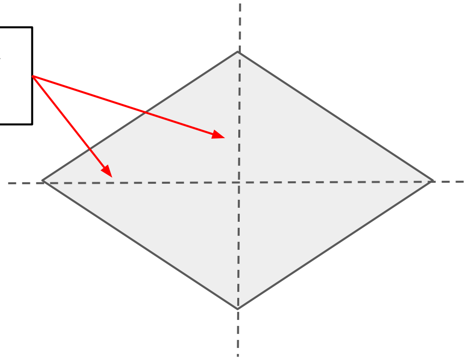


Notice: $f(x) = sin\ \Theta = cos\ (\Theta - \pi/2)$ What other patterns do you see?

Line Symmetry (Symmetric about a line)

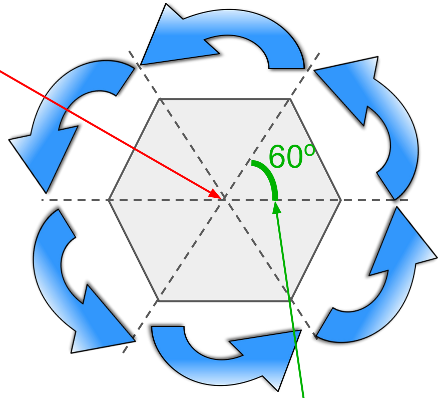
Line of symmetry





Rotational Symmetry (Symmetric about a point)

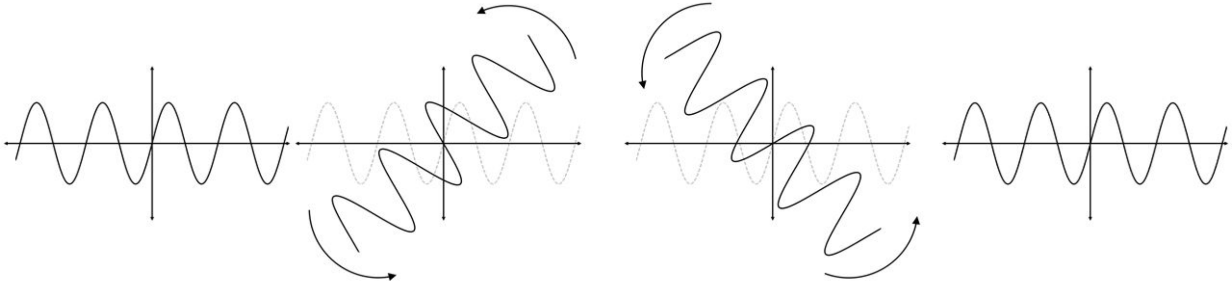
Center of rotation



Angle of rotation

Example: sine function has rotational symmetry.

Sine function rotates 180° around the point (0,0).

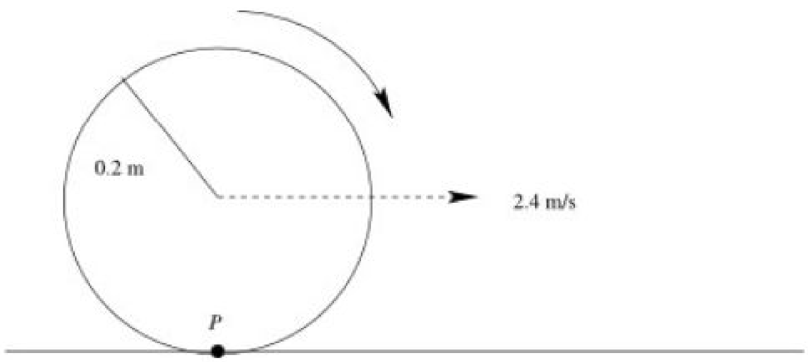


Illustrative Mathematics

F-TF, F-IF As the Wheel Turns

Task

A wheel of radius 0.2 meters begins to move along a flat surface so that the center of the wheel moves forward at a constant speed of 2.4 meters per second. At the moment the wheel begins to turn, a marked point P on the wheel is touching the flat surface.



a. Write an algebraic expression for the function y that gives the height (in meters) of the point P, measured from the flat surface, as a function of t, the number of seconds after the wheel begins moving.

b. Sketch a graph of the function y for $t > 0$. What do you notice about the graph? Explain your observations in terms of the real-world context given in this problem.

c. We define the horizontal position of the point P to be the number of meters the point has traveled forward from its starting position, disregarding any vertical movement the point has made. Write an algebraic expression for the function x that gives the horizontal position (in meters) of the point P as a function of t, the number of seconds after the wheel begins moving.

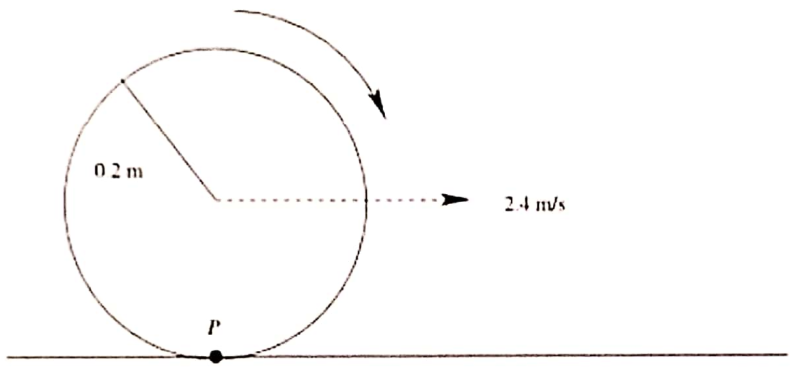
d. Sketch a graph of the function x for $t > 0$. Is there a time when the point P is moving backwards? Use your graph to justify your answer.

Illustrative Mathematics

F-TF, F-IF As the Wheel Turns

Task

A wheel of radius 0.2 meters begins to move along a flat surface so that the center of the wheel moves forward at a constant speed of 2.4 meters per second. At the moment the wheel begins to turn, a marked point P on the wheel is touching the flat surface.



equation y(t)
a. Write an ~~algebraic expression~~ for the function y that gives the height (in meters) of the point P, ~~measured from the flat surface~~, as a function of t, the number of seconds after the wheel begins moving.

see
b. Sketch a graph of the function y for $t > 0$. What do you notice about the graph? Explain your observations in terms of the real-world context given in this problem.
Explain what the graph tells you about the wheel in real life.

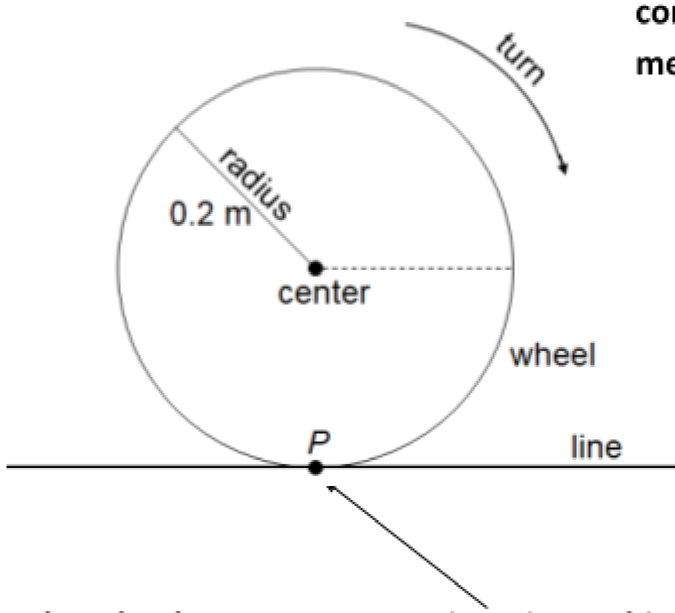
c. We define the horizontal position of the point P to be the number of meters the point has traveled forward from its starting position, ~~disregarding any vertical movement the point has made~~. Write an ~~algebraic expression~~ equation for the function x x(t) that gives the horizontal position (in meters) of the point P as a function of t, the number of seconds after the wheel begins moving.

d. Sketch a graph of the function x for $t > 0$. Is there a time when the point P is moving backwards? Use your graph to ~~justify your answer~~.
explain why you answered yes or no.

F-TF, F-IF As the Wheel Turns

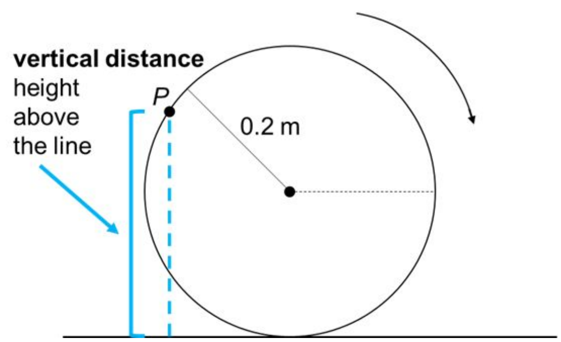
1. The radius of the wheel is 0.2 meters.

2. The wheel starts to turn. The center is moving at a constant speed of 2.4 meters per second.



3. When the wheel starts to turn, point *P* is touching the line.

QUESTIONS:



a) *y* is the height (in meters) of point *P* above the line.

***t* is the time (in seconds) after the wheel starts to turn.**

Write a function *y*(*t*).

b) Make a graph of the function *y(t)* when *t* > 0.

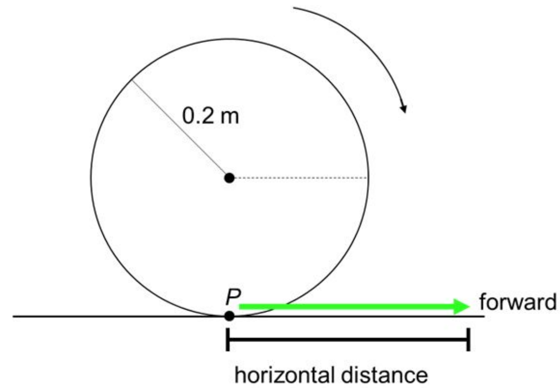
What do you see in the graph?

Explain what the graph tells you about the wheel in real life.

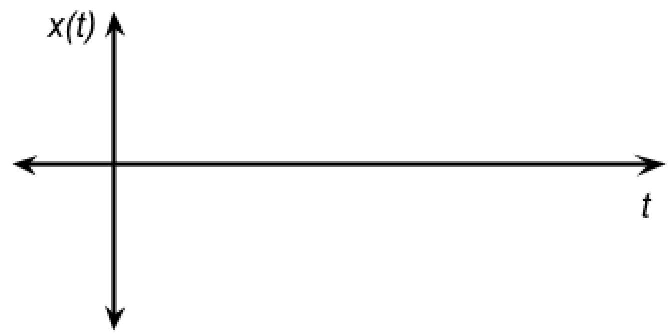
c) ***x*** **is the horizontal distance (in meters) that point *P* moves forward.**

t **is the number of seconds after the wheel starts to turn.**

Write a function *x(t)*.

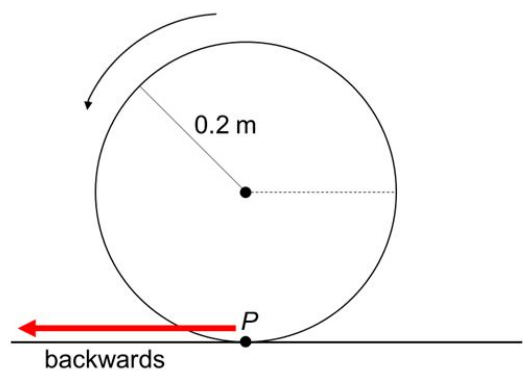


d) **Make a graph of the function *x(t)* when *t* > 0.**

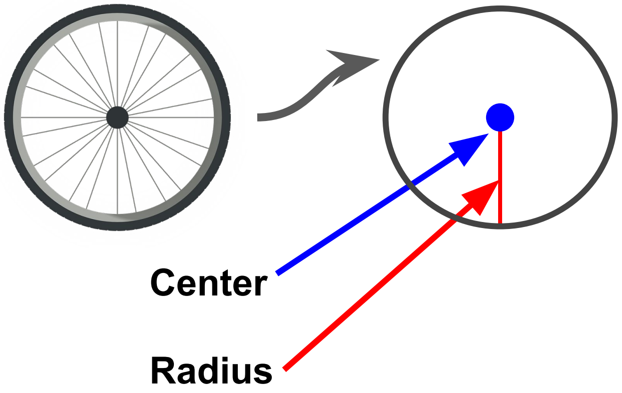


Is there a time when point *P* is moving backwards?

What do you see on the graph to show that the answer is yes or no?

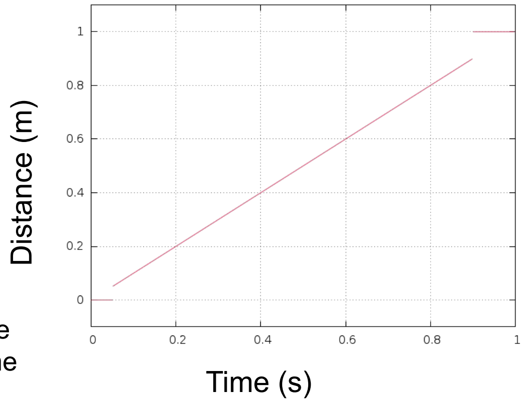


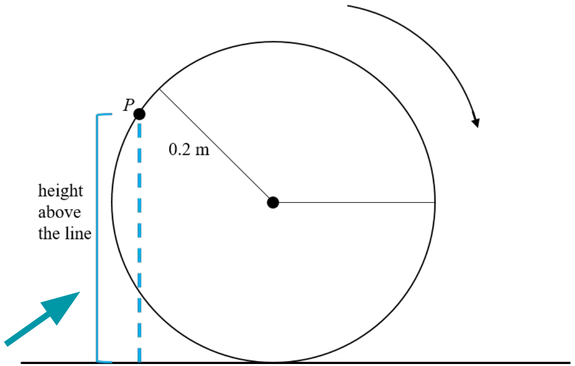
Wheel



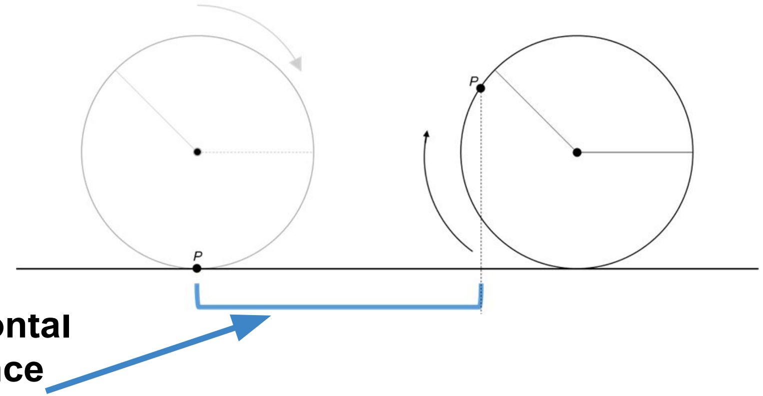
Constant Speed

traveling the same distance
for the same amount of time

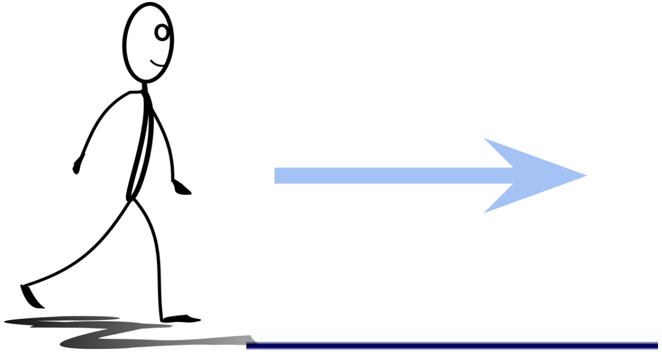
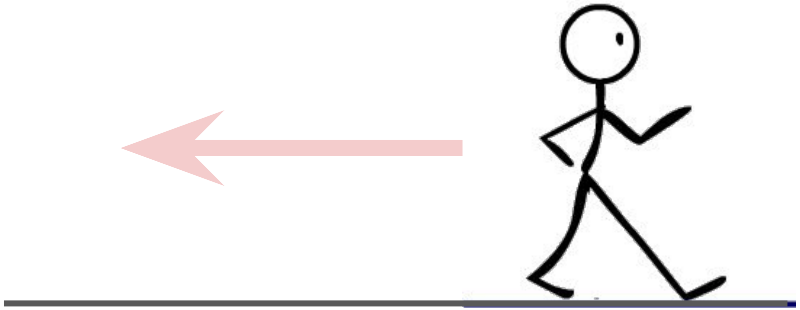


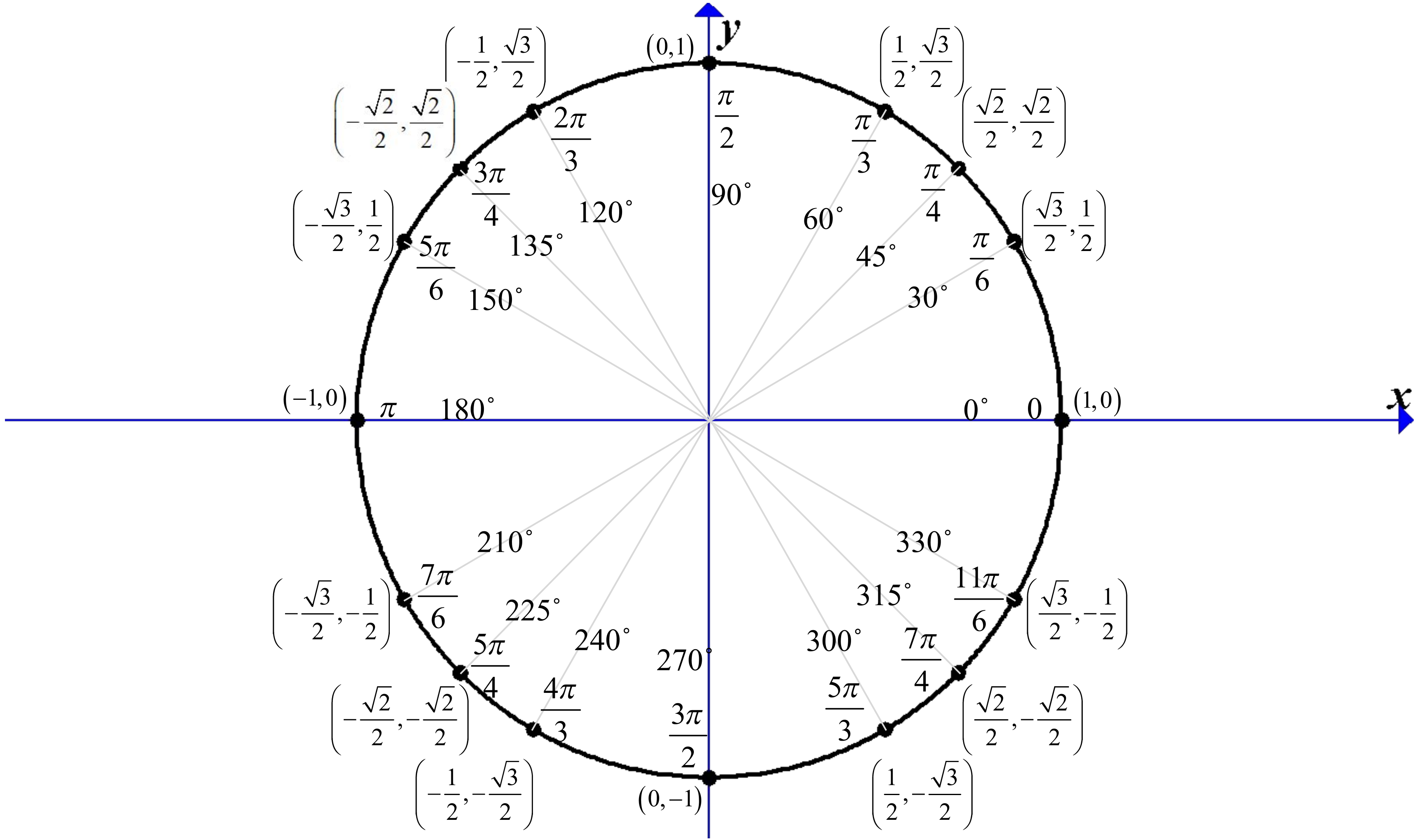


Vertical Distance
(Height above the line)

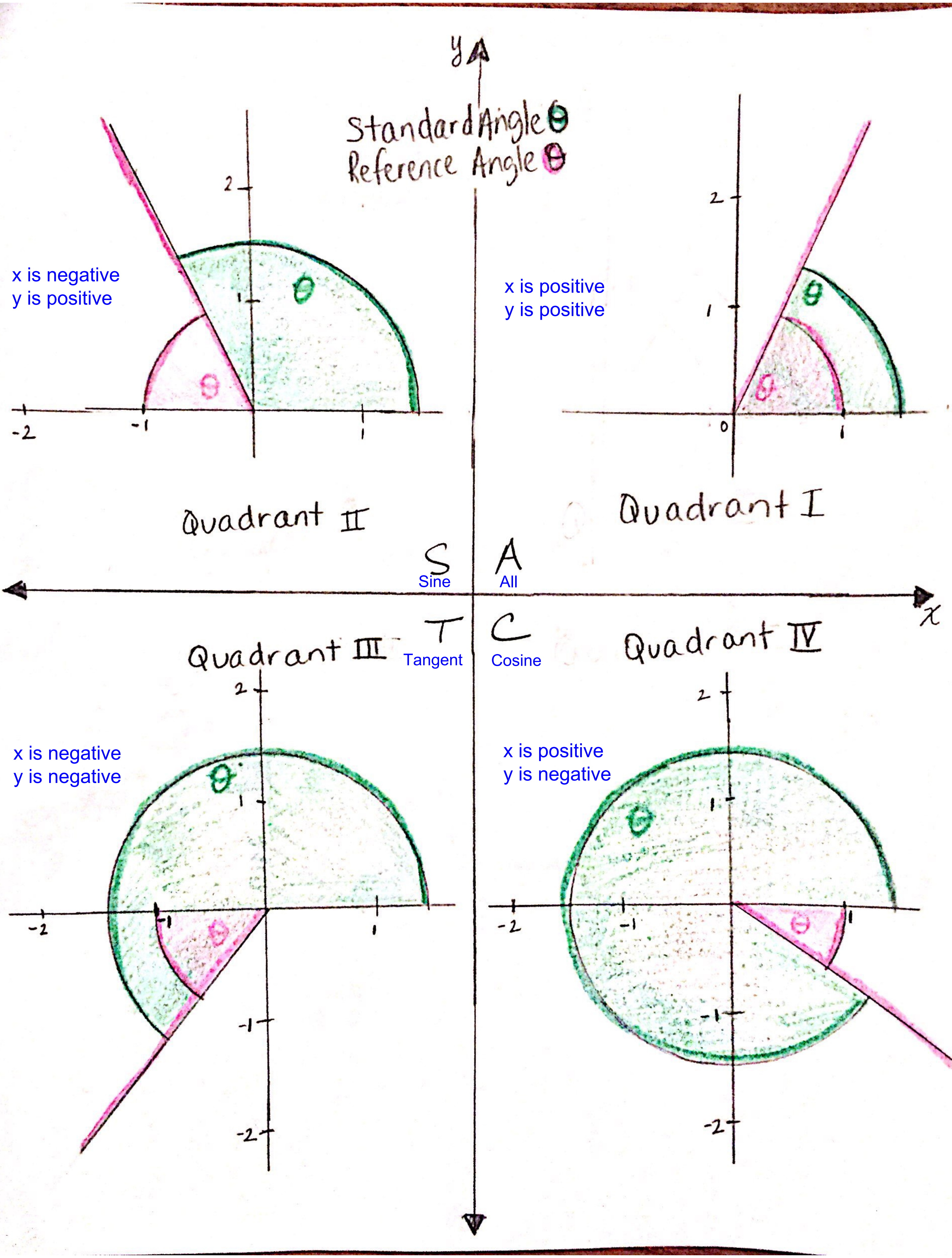


Horizontal Distance

Forward	**Backward**
	



y
x
(0,1)
$\frac{\pi}{2}$
90°
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{3}$
60°
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{4}$
45°
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{\pi}{6}$
30°
(1,0)
0
0°
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\frac{2\pi}{3}$
120°
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
135°
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{5\pi}{6}$
150°
(−1,0)
π
180°
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\frac{7\pi}{6}$
210°
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\frac{5\pi}{4}$
225°
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\frac{4\pi}{3}$
240°
(0,−1)
$\frac{3\pi}{2}$
270°
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\frac{5\pi}{3}$
300°
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\frac{7\pi}{4}$
315°
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\frac{11\pi}{6}$
330°


y
Standard Angle θ
Reference Angle θ
x is negative
y is positive
Quadrant II
x is positive
y is positive
Quadrant I
S
Sine
A
All
T
Tangent
C
Cosine
Quadrant III
x is negative
y is negative
Quadrant IV
x is positive
y is negative
x