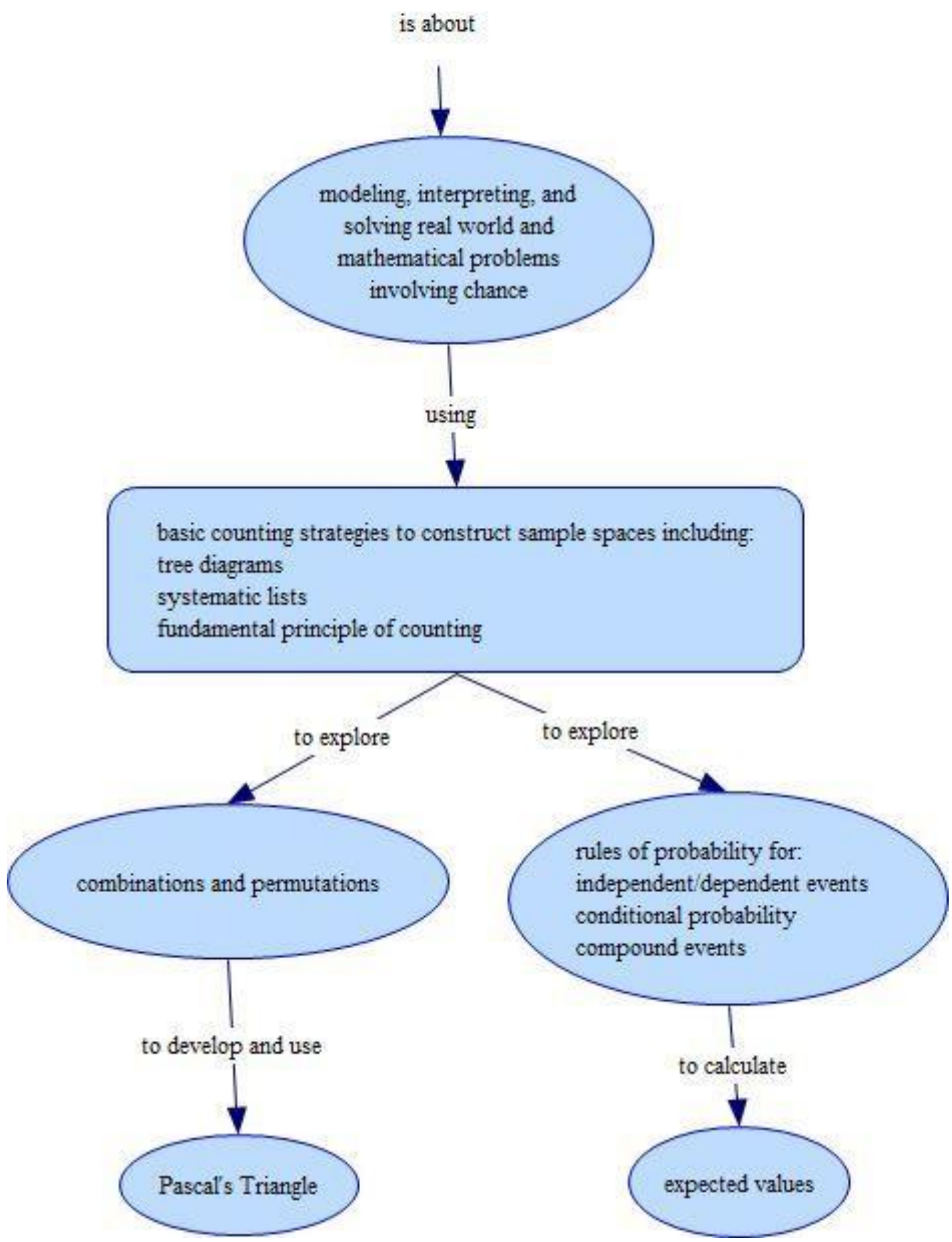
Common Core Initiative

Overarching Questions and Enduring Understandings

How can the ideas of independence and conditional probability, along with expected value, be used to evaluate the outcomes of decisions in a variety of contexts?

Graphic Organizer



Unit Abstract

Although students have had informal experiences with probability in elementary school, their first formal introduction to the study of probability begins in seventh grade. Students conducted experiments or simulations and learned about calculating empirical (experimental) probabilities from their results. They made comparisons of empirical and theoretical probabilities and used theoretical probabilities to predict relative frequencies for a simple chance event. Students learned to set up probability models from a theoretical perspective (such as rolling two dice) or modeling a situation with or without technology and using the empirical data as an estimate of the theoretical outcomes. With compound events, students used tree diagrams, organized lists, or tables to generate the sample space for the event. They use the sample space to calculate the probability of a particular event occurring.

At the high school level, students will extend their knowledge of probability and use their previous experience with simulations as a basis for more complicated ideas. Students will use their experience in making sample spaces in middle school and extend the use of these sample spaces to determine the probability of an event occurring to including the probability of event A and event B occurring, of event A or event B occurring; and the probability of an event not occurring. Students will study a variety of problems in order to develop an understanding of independent and dependent events. They learn that the second event has not been influenced by what occurred in the first event when the two events are independent. The idea of conditional probability, i.e. finding the probability of one event occurring given that another event has already occurred, is studied in this unit and can be used as another way to view independence. In terms of probability notation, conditional probability can be expressed as P (A/B) = P (A and B)/P (B). In determining independence of events A and B, when P (A/B) = P (A) and P (B/A) = P (B), then A and B are independent. Another way to determine independence is through the construction of a two-way frequency table, used when two categories are associated with each object being classified. After all of these calculations, it is important to be able to recognize and understand conditional probability and independence and be able to explain these concepts in everyday language.

Following exploring and understanding the ideas of conditional probability and independence, students are introduced to the rules of probability. These include the Addition Rule P (A or B) = P (A) + P (B) – P (A and B), and the Multiplication Rule P (A and B) = P (A) P (B/A). They should be able to use and interpret these rules. Additionally the use of permutations and combinations will help in determining probabilities and solving problems. Use of Pascal's Triangle is particularly helpful in determining combinations.

When using applications of probability to solve problems, often a numerical quantity is more useful than a description of possible outcomes. Graphing a probability distribution gives a different perspective. This is where the expected value is introduced and interpreted as the mean of the probability distribution. Students will develop a probability distribution from a sample space in which the probabilities were assigned either theoretically or empirically. They will then calculate the expected value.

The use of expected values and probabilities are widely used to solve problems and evaluate decisions. Probabilities can be used to weigh decisions depending on the probability of outcomes. For example how much a company charges for an extended warranty depends on the cost of repairing or replacing an item and the probability that that item will fail. Games of chance depend on the ideas of expected value. Analyzing decisions and strategies involve probability concepts. Students should recognize the wide impact that probability can have on decisions they make.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Statistics/Probability

Conditional Probability & the Rules of Probability

HSS-CP.A. Understand independence and conditional probability and use them to interpret data

- HSS-CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions,

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***

intersections, or complements of other events ("or," "and," "not"). • HSS-CP.A.2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. • HSS-CP.A.3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. • HSS-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.* • HSS-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.* HSS-CP.B. Use the rules of probability to compute probabilities of compound events in a uniform probability model • HSS-CP.B.6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. • HSS-CP.B.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. • HSS-CP.B.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. • HSS-CP.B.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **Using Probability to Make Decisions** HSS-MD.A. Calculate expected values and use them to solve problems • HSS-MD.A.1. (+) Define a random variable for a	2. ***The standards contain content that is developed and/or utilized across multiple units.*** Modified For this Unit n/a Developed and/or Utilized Across Multiple Units n/a

quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
• HSS-MD.A.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
• HSS-MD.A.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
• HSS-MD.A.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
HSS-MD.B. Use probability to evaluate outcomes of decisions
• HSS-MD.B.5. (+)Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
  ○ HSS-MD.B.5a. Find the expected payoff for a game of chance.
For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
  ○ HSS-MD.B.5b. Evaluate and compare strategies on the basis of expected values.
For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
• HSS-MD.B.6. (+)Use probabilities to make fair decisions
(e.g., drawing by lots, using a random number generator).
• HSS-MD.B.7.(+) Analyze decisions and strategies using probability concepts
(e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).	

Essential/Focus Questions 1. How can you generate the numerical values of Pascal's Triangle? 2. How do you recognize when to use conditional probability rules? 3. What is the difference between permutations and	**Key Concepts** Pascal's triangle, and its connections to combinations Permutation P(n,k)=n!/(n-k)! Combination C(n,k)=n!/[(n-k)!k!] Fundamental Principle of Counting

combinations? Give an example of a situation where each would be used. 4. If the probability of an event occurring is p, what is the probability of that event not occurring? Explain why your answer makes sense. 5. What is the meaning of expected value?	Tree Diagram Sample space Probability Distribution Independent events Multiplication Rule P(A and B)=P(A) • P(B) Area Model Dependent events Mutually exclusive events Addition Rules for Mutually Exclusive events Compound events Complementary events Conditional probability P(A|B)=P(A and B)/P(B) Applications of probability to real-world situations Simulation Law of Large Numbers Expected Value Two-way frequency table
Assessment Tasks Assessment Overview A Game of Dice Sheet	**Intellectual Processes** **Standards of Mathematical Practice** *Students will have opportunities to:* • **make sense of problems and persevere in solving them** using probability that arise in mathematics and in other contents; and • **use appropriate technology tools strategically** to explore and deepen understanding of probability concepts.
Lesson Sequence Lesson Overview Professional Learning Tasks-Student work sample-Questions Probability Highlight Lesson, Conditional Probability	**Resources** Unit Resources