ELD STANDARD 3: The Language of Mathematics	MAISA Algebra 2, Unit 8, Probability

CONNECTIONS; Michigan Academic State Standards for Mathematics

EXAMPLE CONTEXT FOR LANGUAGE USAGE: The task for this strand is from Illustrative Mathematics, located at https://www.illustrativemathematics.org/content-standards/tasks/1217. Here, students decide whether playing a lottery game is profitable in the long run. Students will begin by analyzing a lottery game with a partner or small group. In order to build understanding about the theoretical expected values, students may design a simulation where they use the probabilities to compare experimental and theoretical results. (A tool for designing spinners and recording the probabilities can be found at: https://www.nctm.org/adjustablespinner/). Students should decide how much they win or lose for each region of the spinner, and track their profit / loss as they spin. After experimenting, students should calculate the expected value and compare results.

As they examine intermittent results and the overall expected value, students discuss the mathematics together. Teachers should intentionally plan for opportunities for all students to produce language whether it is in the small groups or whole group summary. The task is used as a speaking task but additional scaffolds/supports are needed to differentiate for the reading/writing aspect. The corresponding CCSS is HSS-MD.B.5a. Find the expected payoff for a game of chance. (For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.)

Note that the words in the required and suggested word lists below are specific to the context of the example problem. To apply this support to different contexts and other problems, the teacher and students should co-create the needed required/suggested words lists as new contexts are introduced.

COGNITIVE FUNCTION: Students at all levels of English language proficiency will use probability to **EXPLAIN** whether a given situation is favorable or unfavorable.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Speaking	Explain using simple sentences, including words specific to the context, whether a given situation is favorable or unfavorable using sentence frames with choices and an illustrated reference sheet and pointing to written calculations. E.g., Phil _____(will / will not) earn money after playing the game many times. There is a _____(%) chance of winning $___ . There is a _____ (%) chance of losing $____. The expected value is_____. There is an average _____ (loss/gain) of $_____ per game.	Explain using simple sentences, including words specific to the context, whether a given situation is favorable or unfavorable using sentence frames with choices and an illustrated reference sheet. E.g., Phil _____(will / will not) earn money after playing the game many times. There is a _____(%) chance of winning $___ . There is a _____ (%) chance of losing $____. The expected value is_____. There is an average _____ (loss/gain) of $_____ per game.	Explain using complete sentences, including words specific to the context, whether a given situation is favorable or unfavorable using a suggested word list (e.g., not profitable/profitable, winning, losing, loss/gain, lottery game, expected value, average, chance) and an illustrated reference sheet. E.g., "I think Phil should not play the game many times. There is a 60% chance of winning $5. The expected win is $3 (5 x 0.60 = 3). There is a 40% chance of losing $10. The expected loss is $4 (-10 x 0.4 = -4). The expected value is -1. That is an average loss of $1 per game."	Explain using compound and/or complex sentences, including words specific to the context, whether a given situation is favorable or unfavorable using a suggested word list specific to the context (e.g., not profitable/profitable, winning, losing, loss/gain, lottery game, expected value, average, chance). E.g., "Phil should not play the game many times because the lottery game is not profitable in the long run. Since there is a 60% chance of winning $5, the expected win would be $3 (5 x 0.60 = 3). With a 40% chance of losing $10, the expected loss would be $4 (-10 x 0.4 = -4). This means the expected value is -1, meaning there is	Explain using compound and/or complex sentences, including words specific to the context, whether a given situation is favorable or unfavorable using a required word list specific to the context (e.g., not profitable/profitable, winning, losing, loss/gain, lottery game, expected value, average, chance). E.g., "Phil should not play the game many times because the lottery game is not profitable in the long run. Since there is a 60% chance of winning $5, the expected win would be $3 (5 x 0.60 = 3). With a 40% chance of losing $10, the expected loss would be $4 (-10 x 0.4 = -4). This	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued				an average loss of $1 per game. So, playing many times would not result in a long term gain."	means the expected value is -1, meaning there is an average loss of $1 per game. So, playing many times would not result in a long term gain."	

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EXAMPLE CONTEXT FOR LANGUAGE USAGE: This task is based on a practice SAT question. The original question can be found on page 47 of the pdf file of SAT Practice Test 7 at https://collegereadiness.collegeboard.org/sat/practice/full-length-practice-tests. However, for the purposes of this strand the task (available in the supports) has been modified and can be found in the supports for this unit. In this activity, the reading task involves reading a table and the writing task involves writing questions related to the table.

In general, SAT questions of this nature are multiple choice, where the student is choosing the best answer to a pre-determined question. This requires being able to read and analyze the information in a table. As students prepare for a standardized assessment, it is important to provide appropriate scaffolds to engage them in mathematical sense-making. As students become more proficient in the language, teachers should gradually remove the scaffolds and monitor their growth, so that students do not become too reliant. It is important to note that scaffolds like those illustrated below (e.g., illustrated tasks, sentence frames, anchor charts, etc.) are not provided on standardized assessments.

Teachers can ask additional clarifying questions to support mathematical reasoning as needed and gradually ask fewer questions as students build the habits of mind to ask themselves those questions. Suggestions include: What do the numerator and denominator represent? Where do you see the numbers from the fraction in the table? What clues can the row and column headings give you about the words to use? (Students might use highlighters to give themselves a visual cue relating the numerical and verbal representations of the probabilities.)

COGNITIVE FUNCTION: Students at all levels of English language proficiency **ANALYZE** information in a table in order to **CREATE** questions based on that set of data that result in required answers.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
Reading & Writing	Analyze information in a table in order to create questions based on that set of data that result in the given answers using an illustrated task sheet, a unit anchor chart, stems to support reasoning, and a sentence frame for writing, while working with a partner. Stems to support reasoning as students read the table: The numerator represents... The denominator represents... The fraction represents... Frame for writing: What fraction of _____ are _____?	Analyze information in a table in order to create questions based on that set of data that result in the given answers using an illustrated task sheet, a unit anchor chart, stems to support reasoning, and a sentence frame for writing, while working with a partner. Stems to support reasoning as students read the table: The numerator represents... The denominator represents... The fraction represents... Frame for writing: What fraction of _____ are _____?	Analyze information in a table in order to create questions based on that set of data that result in the given answers using an illustrated reference sheet, a unit anchor chart, and stems to support reasoning, while working with a partner. Stems to support reasoning as students read the table: The numerator represents... The denominator represents... The fraction represents... E.g., A. "Of all the animals at the pet care facility, what fraction are dogs who are fed dry food? B. "What fraction of the dogs are fed only dry food?"	Analyze information in a table in order to create questions based on that set of data that result in the given answers using a unit anchor chart while working with a partner. E.g., A. "Of all the animals at the pet care facility, what fraction are dogs who are fed dry food? B. "What fraction of the dogs are fed only dry food?" C. "What fraction of the animals at the pet care facility are fed only dry food?" D. "What fraction of the animals who are fed dry food are dogs?"	Analyze information in a table in order to create questions based on that set of data that result in the given answers using a unit anchor chart and while working with a partner. E.g., A. "Of all the animals at the pet care facility, what fraction are dogs who are fed dry food? B. "What fraction of the dogs are fed only dry food?" C. "What fraction of the animals at the pet care facility are fed only dry food?" D. "What fraction of the animals who are fed dry food are dogs?"	

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 Reaching
continued			C. "What fraction of the animals at the pet care facility are fed only dry food?" D. "What fraction of the animals who are fed dry food are dogs?"			

Two-Way Table Relative Frequencies

Question 1: Shawnta likes chocolate. What is the probability that she also likes vanilla?

Answer 1:

$$P(B \mid A) = \frac{\text{\# who like both}}{\text{\# who like chocolate}} = \frac{60}{98} \approx 61\%$$

Probability that Shawnta likes vanilla if we know she likes chocolate.

Question 2: Devin likes vanilla. What is the probability that he also likes chocolate?

Answer 2:

$$P(A \mid B) = \frac{\text{\# who like both}}{\text{\# who like vanilla}} = \frac{60}{90} \approx 67\%$$

Probability that Devin likes chocolate if we know he likes vanilla.

		Condition B (likes vanilla)		
		Yes	No	Total
Condition A (likes chocolate)	Yes	60	38	98
	No	30	21	51
	Total	90	59	149

Number of people who like both chocolate **and** vanilla

Number of people who like vanilla

Number of people who like chocolate

Average

Examples:

Leveling Out
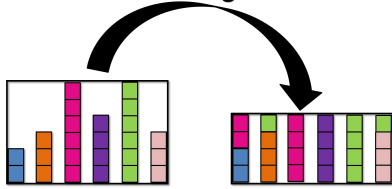
$\frac{5+7}{2} = 6$ $\frac{2+10+5}{3} = 5.\overline{6}$

Lottery Game

PICK 2 3 4   Win up to $50,000 Twice a Day!
Thousands of Daily Winners!

Playing the lottery many times

Wins	Loss	Profit
5	10	-$13

Profitable

Not profitable

Expected Value

1	1
2	3

expected value

$= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 3$
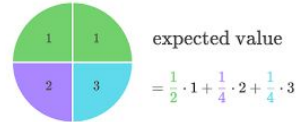
Theoretical Graph

+ $1
- $6 $3

Experimental Graph

Color	Count
Blue	111
Yellow	46
Cyan	43
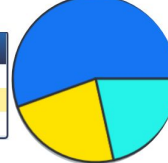
$111 \cdot \$1 - 46 \cdot \$6 + 43 \cdot \$3 = -\$36.00$

Chance

%

Winning	Losing
$ + $ + $	$ - $ - $

Gain	Loss

Feeding Information for Boarded Pets

	Fed only dry food	Fed both wet and dry food	Total
Cats	5	11	16
Dogs	2	23	25
Total	7	34	41

The table above shows the kinds of foods that are fed to the cats and dogs currently boarded at a pet care facility. Use the data to **create a question** that will generate **each** of the following **answers**:

Answer:	Write a question that matches the answer:
A. $\dfrac{2}{41}$	
B. $\dfrac{2}{25}$	
C. $\dfrac{7}{41}$	
D. $\dfrac{2}{7}$	
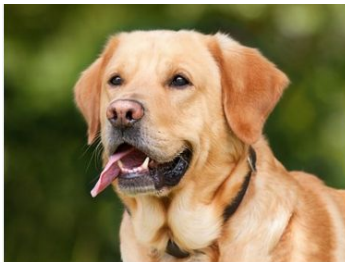
Cat	Dog	Boarded Pet

Dry Food	Wet Food	
		$$\frac{8 \text{ Numerator}}{21}$$ Denominator
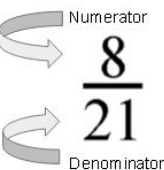
Feeding Information for Boarded Pets

	Fed only dry food	Fed both wet and dry food	Total
Cats	5	11	16
Dogs	2	23	25
Total	7	34	41

The table above shows the kinds of foods that are fed to the cats and dogs currently boarded at a pet care facility. Use the data to **create a question** that will generate **each** of the following **answers**:

Numerator

$$\frac{8}{21}$$

Denominator

Probability as a fraction	Create a question that will generate each of the given fraction
$\frac{2}{41}$	
$\frac{2}{25}$	
$\frac{7}{41}$	
$\frac{2}{7}$	