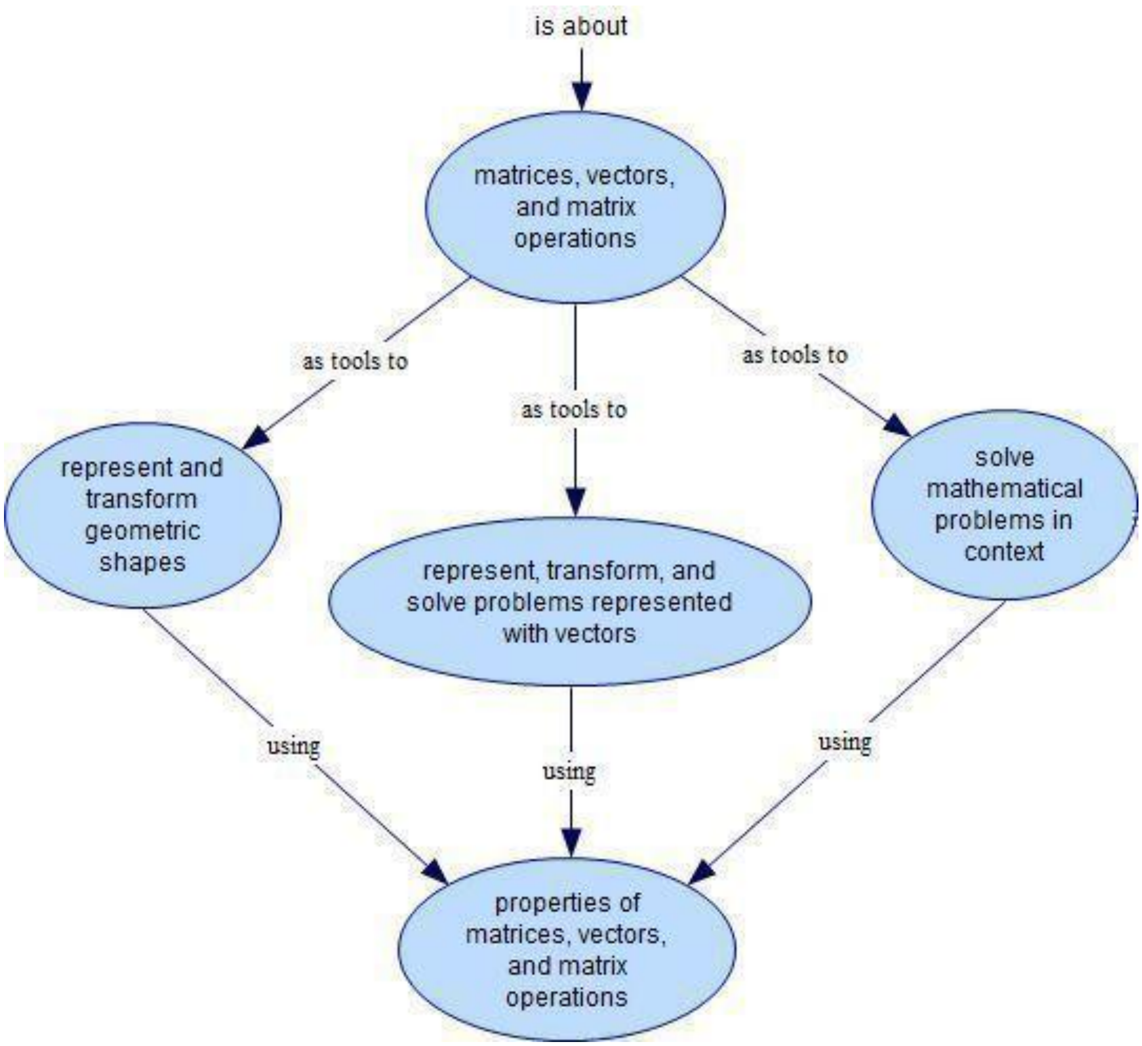
Common Core Initiative

Overarching Questions and Enduring Understandings

How can matrices and vectors be used to solve problems in mathematics and other related fields?

Graphic Organizer



Unit Abstract

In this unit, students first use matrices as a way to organize and display data. After constructing matrices, students can then use the properties of matrices to analyze data and solve a variety of problems from different contexts. Students, without the use of technology, can attain understanding of basic matrix operations. However, one important use of matrices is to solve systems of equations. In doing this, the use of technology to find matrix inverses significantly simplifies the process. This can be accomplished by the use of a graphing calculator or computer software. Using matrices to solve systems of equations provides students tools to solve linear programming problem situations that can be very complex.

Another important application for matrices is the representation of geometric shapes in matrix form. These matrix representations can be used in combination with matrices representing various transformations in the plane to show movement and animation of objects mathematically.

Matrices are often used to solve problems involving vectors. Because students have studied matrices in this unit, it makes sense to include the study of vectors and using matrices to help solve these problems. Students should understand the components of vectors, how to add and subtract vectors, and how to multiply a vector by a scalar. Problems related to vectors are commonly used in solving physics problems. Although vectors are designated as STEM standards, it would be worthwhile for exposure and interdisciplinary cooperation to include vector problems in instruction.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Number/Quantity

Vector & Matrix Quantities

HSN-VM.A. Represent and model with vector quantities.

- HSN-VM.A.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
- HSN-VM.A.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- HSN-VM.A.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

HSN-VM.B. Perform operations on vectors.

- HSN-VM.B.4. (+) Add and subtract vectors.
  - HSN-VM.B.4a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - HSN-VM.B.4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - HSN-VM.B.4c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- HSN-VM.B.5. (+) Multiply a vector by a scalar.
  - HSN-VM.B.5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(v?, v?) = (cv?, cv?).
  - HSN-VM.B.5b. Compute the magnitude of a

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit
n/a

Developed and/or Utilized Across Multiple Units
n/a

scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ? 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). **HSN-VM.C. Perform operations on matrices and use matrices in applications.** • HSN-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. • HSN-VM.C.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. • HSN-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions. • HSN-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. • HSN-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. • HSN-VM.C.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. • HSN-VM.C.12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. **High School: Algebra** **Reasoning with Equations & Inequalities** **HSA-REI.C. Solve systems of equations.** • HSA-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. • HSA-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). © Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.	
Essential/Focus Questions 1. How can you tell whether two matrices can be multiplied together? 2. What process is used to solve a system of linear equations using matrices? 3. How can matrices be used to represent a	**Key Concepts** associative property commutative property Cramer's Rule determinant dimension element

polygon? 4. Given a polygon represented in matrix form, how can you use a matrix to rotate the polygon 180^0? 5. How can two vectors be added together? 6. How do you represent a vector by using a matrix?	identity matrix inverse matrix linear programming matrix operations multiplication by a scalar solving systems of equations transformation matrices vector components magnitude direction initial point terminal point add and subtract vectors resultant parallelogram rule scalar scalar multiplication
Assessment Tasks Assessment Overview Matrices Assessment.doc Matrices Assessment Key.doc Matrix Re-engagement Student.docx Matrix Re-engagement teacher_notes1.docx	**Intellectual Processes** **Standards for Mathematical Practice** *Students will have opportunities to:* • **attend to precision:** create and use matrix representations to organize, record, and communicate mathematical ideas; • **look for and make use of structure:** understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and • **reason abstractly and quantitatively:** apply and adapt a variety of appropriate strategies to solve problems using matrices.
Lesson Sequence Lesson Overview Numbers Lesson Description-Parabolic Food Fight Professional Learning Task - Teacher Reflection Video: Using Reflection to Plan the Lesson	**Resources** Unit Resources

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