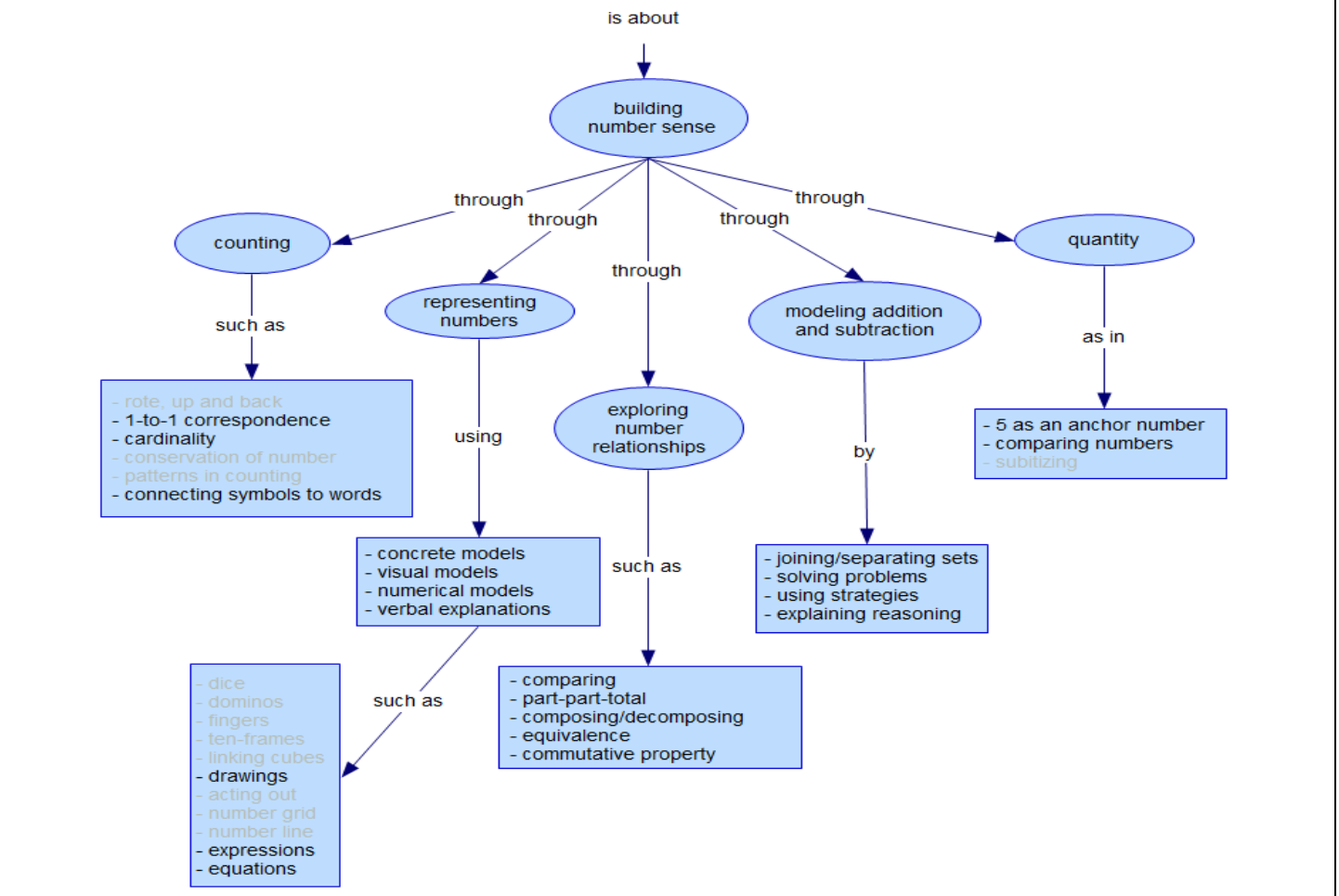
Kindergarten: Mathematics
Highlight Lesson
Unit 2: Exploring Number

Overarching Question:
What do you know about the number five?

Previous Unit:	**This Unit:**	**Next Unit:**
Exploring Attributes and Shape Part I	Exploring Number	Exploring Measurement



Questions to Focus Assessment and Instruction:

1. How do numbers relate to each other?
2. How do we use numbers to count?
3. How do we use numbers to compare?
4. Why is five a special/helpful number?
5. What can help us to recognize numbers instantly and accurately without counting (subitize)?
6. What does it mean to break apart (decompose) and to combine (compose) numbers?

Intellectual Processes
Standards for Mathematical Practice

Students will have opportunities to:

- **make sense of problems** that focus on number relationships **and persevere in solving them;**
- **reason abstractly and quantitatively** when breaking numbers apart, comparing numbers, adding and subtracting,
- **model with** dot cards, five frames; and
- **look for and make use of structure** of numbers to five.

Key Concepts:

quantity	counting	comparing numbers	part-part-total
one-to-one correspondence	compose/decompose	equivalence	commutative property
instant recognition (subitizing)	5 as an anchor number	representing numbers	compensation

Lesson Abstract:
Students have been working with multiple representations to internalize an understanding of "five", which is an anchor for later developing an understanding of the anchor "ten." In this lesson, students will work with a partner to solve a word problem by building all the 2-addend combinations for five. They will work to *discover* the pattern, 0+5, 1+4, 2+3, 3+2, 4+1, 5+0, laying the foundation of the important numerical concepts of part-part-total, composing/decomposing numbers, equivalence, compensation (adjusting the numbers in an expression while maintaining the value, e.g., 6 + 4 = 5 + 5; 9 + 6 = 10 + 5), and the commutative property for addition.

This lesson should be repeated throughout the year as students become ready to work with larger numbers to ten. Continue using these trains for representing number pairs. In addition, beginning to use a ten-frame helps student see these numbers in relationship to the anchor number "ten." When filling in a ten-frame, always begin in the upper left corner, fill in the top row (anchor number 5), then begin at the left of the second row and fill in the rest of the target number. This both reinforces the direction in which students are learning to read and introduces a common protocol for working with ten-frames.

Common Core Standards

Kindergarten, Counting and Cardinality

K.CC.A Know number names and the count sequence.
- K.CC.A.1 Count to 100 (30) by ones and by tens.
- K.CC.A.3 Write numbers from 0 to 20 (10). Represent a number of objects with a written numeral 0-20 (10) (with 0 representing a count of no objects).

K.CC.B Count to tell the number of objects.
- K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
  - K.CC.B.4.a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
  - K.CC.B.4.b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  - K.CC.B.4.c. Understand that each successive number name refers to a quantity that is one larger.
- K.CC.B.5. Count to answer "how many?" questions about as many as 20 (10) things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20 (10), count out that many objects.

Kindergarten, Operations and Algebraic Thinking

K.OA.A Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
- K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- K.OA.A.3 Decompose numbers less than or equal to 10 (5) into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
- K.OA.A.4. For any number from 1 to 9 [4], find the number that makes 10 [5] when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- K.OA.A.5 Fluently add and subtract within 5.

Instructional Resources
- Linking cubes – enough for pairs of students to have 15 red cubes and 15 green cubes
- A way to display and arrange the set of solutions after students have completed the task. For example,
  - transparent linking cubes or counters to project on an overhead projector,
  - linking cubes on a document camera, or
  - magnetic models of linking cubes or green and red tiles on a magnetic board.
- Student Handouts "Creating Fives" and/or "Creating Sixes" (a Student Handout found in the lesson resources)

Sequence of Lesson Activities

Lesson Title: "Five-ness": Five as an Anchor Number

One Class Period (See NOTE under Summary: Sharing and Discussing the Task)

Advanced Preparation:	
▪ Collect sets of linking cubes so that each pair of students will have 15 red cubes and 15* green cubes. Students will work together to create one set of the combinations. For one set they will need a total of 30 cubes (15 of each color). You may need to borrow cubes from another classroom for this lesson. Any other two colors are fine, but all pairs of students need to have the same two colors. This is important for the group discussion at the end of the lesson. Red and green match the story (about grapes) that will launch this activity. ▪ If you find students need to work independently, you will need twice as many linking cubes. ▪ Double-sided counters (e.g., one side yellow, one side red) could be used for this activity when it is at centers. For this whole class activity, linking cubes are best because they are so easy for students to manipulate – including turn around (commutative property). ▪ Determine which students will be partnered together for this activity.	
Selecting and Setting Up a Mathematical Task	
Objective: By the end of this lesson what do you want your students to understand, know, and be able to do?	Students will: • make all six additive combinations for five (complements of five), i.e., as the number of one-color cube increases, the number of the other color cube decreases. To meet the objective, students <u>*do not*</u> need to display the cube "trains" in order, i.e., 0+5, 1+4, 2+3, 3+2, 4+1, 5+0, as long as they create all six combinations. • show cube trains that exhibit the commutative property. • explain that when taking away one color in exchange for adding another of the second color, the total number of cubes does not change (compensation) • translate between a context and its concrete and pictorial forms and its numerical expression.
In what ways does the task build on students' previous knowledge?	Students may have been working with groups to five in several contexts: • working with various collections of common objects to create sets of 5 objects, e.g., buttons, rocks, bottle caps, sea shells, toothpicks, beads, craft sticks, linking cubes, colored tiles • matching numeral cards to sets of objects between one and five, e.g., playing Concentration • solving word problems to five • identifying quantities from one to five without counting (subitizing) using quick image activities for dot card arrangements
What questions would you ask to help students access their prior knowledge?	*This month we've been talking a lot about five.* • *What do you know about five?* • *Let's think about all the different ways we have been looking at five.* Students will mention activities such as those listed above. • *Let's play a little game to warm up our brains before we start our lesson.* Say a number 0 – 5. Have students hold up that many fingers. Say the numbers in random order, e.g., *Show me 1; show me 4,* etc.
Launch:	
How will you introduce students to the activity so as to provide access to all students while maintaining the cognitive demands of the task?	(It is assumed that students will have used linking cubes already this year. If not, they will need some free exploration time to "play with" the cubes before starting this lesson.) The following story sets this math activity into a real-life context. • *Last year, Leah brought grapes to class to share for snack. (What are grapes?) Leah brought green grapes and red grapes. She brought enough grapes so that each student could have five. It didn't matter how*

	many of each color students got. The only rule was that they could only have a total of five grapes. Leah didn't have time to count out the grapes, so that each student got the same number of red grapes and green grapes. So some students got more green grapes and some students got more red grapes. After snack, Leah wondered how many different combinations of red and green grapes there were. What do you think? (Have a few students make an estimate.) • *Let's see if we can help Leah figure that out.* Hold up a linking cube train with 2 red and 3 green cubes. • *How many red grapes did this child have?* • *How many green grapes?* (It doesn't matter which color you show first.) • *What was the total numbers of grapes Leah gave this child?* • *So 2 red and 3 green is one of the ways (combinations) to get a total of 5 grapes.* Repeat the same demonstration with 4 + 1. • *You will work with a math partner. One of you will get a set of green linking cubes (to represent green grapes) and your partner will get a set of red linking cubes (to represent red grapes). I want you to work together to see how many different combinations of five you can make.* **NOTE: Do <u>not</u> show them how to set the cube trains up <u>in sequential order</u>, i.e., 0+5, 1+4, 2+3, 3+2, 4+1, 5+0, during this part of the lesson.** This is a much more complex task that students are not likely to come up with on their own at this point. You will demonstrate this to them during your class discussion <u>only</u> to see if they have all combinations. <u>Do not encourage or require them to use this strategy</u>. Prescribing use of this pattern during the explore may limit students to continuing the pattern (doing a rote procedure) without counting and developing the number sense that this activity potentially affords. Eventually, they will discover this pattern on their own.

What will be heard that indicates that the students understood what the task is asking them to do?	• *Why are we making combinations of <u>5</u>?*
	  ○ Because that's how many grapes each student got.
	• *What will you be doing with your partner?*
	  ○ Working together to see how many ways we can make a set of 5 grapes/linking cubes.
	  ○ Putting green and red cubes together.
	  ○ Collecting trains of five cubes.

Supporting Students' Exploration of the Task:	

What questions will be asked to focus students' thinking on the key mathematics ideas?	• *How many grapes should we have all together?*
	• *How do you know that this is five grapes? Will you show me?*
	  ○ I counted. (Watch for one-to-one correspondence and cardinality.)
	  ○ I saw two green, then (pointing at the reds) three, four five. (This student started with 2 and counted on.)
	  ○ I lined it up to this one that I knew was five, and they were the same.
	
	For this lesson, you want students to be able to come up with all the possible combinations for two colors in a way that will have them decompose 5 into two addends. During the free exploration with sets of objects, they often came up with sets of 2 + 1 + 2 (2 red/1 green/2 red) or 1 + 1 + 1 + 1 + 1 (red/green/red/green/red), etc. This story context encourages thinking in ways that group the like-colors together. This is what you want so that students will be able to recognize the compensation pattern (Taking one (1) from one addend and adding it to the other does not change the total) and the commutative property.
	
	• *If you put all of your reds together, how many reds do you have? How would that change your expression?*
	• *How does this set of "grapes" compare to that set?*
	  ○ They both have 5 grapes.
	  ○ This one has more red but less green.

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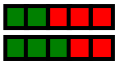
When comparing combinations that elicit the commutative property: How are these combinations similar and different?

They both have 5 grapes.
This link has 2 red and 3 green, but this one has more red and less green.
For partners who have sets that represent the commutative property of addition (i.e., 2+3 = 3+2), teachers might have the following dialogue with a small group. What do you notice about these two combinations? 

One has more green and the other has more red. Tell me more.
One has 2 green and 3 red and the other has 3 green and 2 red.
I lined them up different and saw that in the  first one I had two green. In the second one I had  two red.

Keep these two combinations in your mind. Now, let's look at your combination of 1 green and 4 red. Do you think there might be a related combination? 
After having this conversation, these two students might be able to explain this pattern to the rest of the class.

Do your table partners have any combinations that you don't have? How do you know that their combination is correct? Would you like to share any combinations with them?
How many different combinations have you found?
Have you found all the possible combinations?
How do you know if you've found all the combinations?

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<tr><td>How will you extend the task to provide additional challenge?</td><td>See NOTE above.
After building concrete representations, some students may be ready to record what they have built on paper (pictorial level) and to write a numerical expression (abstract level) below the sets they have created, e.g., 2 + 3. As an aid, provide a page with six outlines of a 5-linking-cube-train. Do not skip the concrete level of representation.
The handout, "Creating Fives", is included in the resources for this lesson. Use the same red and green crayons to model the combinations you built with the linking cubes. Then write an expression below each combination.

Some students may be ready to do this task with 6 or more ("grapes"). "Creating Sixes" is also available as a printable handout for students to record the combinations they build and the related numerical expressions.</td></tr>
<tr><td colspan="2">Summary (Sharing and Discussing the Task):</td></tr>
<tr><td>What specific questions will be asked so that students make connections between the different strategies that are presented?</td><td>NOTE: If you find that this activity will take two days, this is the place to break. Have partners save their collections until the next day and bring the class together as a group for the sharing and discussion.

Gather students together in the class meeting area, having students sit with their partners and bring their constructions with them. Have students put their linking cube rods on the floor in front of them so they can see them all.

• Did anyone create a group with only red grapes?
If nobody came up with this option, just move to the next combination. If a group does show you a rod of five red linking cubes, model this on a document camera, an interactive whiteboard, with magnetic tiles on a magnetic board, or on an overhead projector (e.g., Make a blackline master transparency of "Creating Fives". Color them in as you move through this discussion.)
• Did anyone create a group of 1 green grape and the rest red grapes?
Guide this by displaying 1 green cube below the first red cube in the rod above. If nobody had a rod with 5 red cubes, begin the pattern with this row, with the green cube on the left end.
• How many red grapes do you have in this arrangement?</td></tr>
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Add four red cubes after the green one in your display.

- *So now what do you see here?* (1 green grape and 4 red grapes.)
- *And how many grapes are there altogether?*
- *Did anyone create a group of 2 green grapes?* Display 2 green cubes.
- *How many red grapes do you have in this row?* After a student has shown an arrangement with 2 green cubes and 3 red cubes, add 3 red cubes next to the green cubes in your display.
- *So now what do you see here?* (2 green grapes and 3 red grapes.)
- *How many grapes do you have in all?*

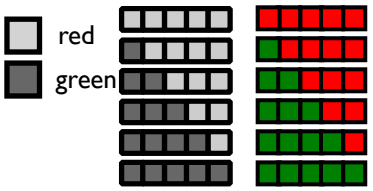
Continue this routine until all six combinations are displayed. If nobody creates an arrangement with 5 red cubes or 5 green cubes, ask,

- *Turn and talk to someone sitting next to you. Is it possible for someone to get all red grapes?*
- *Now talk about where I should put this group? Why? Can someone restate Alison's thinking? Would anyone like to add on?*

Display it with 5 cubes as the top row.

- *Is it possible for someone to get all green grapes?*
- *What would that look like?*
- *Where should I put this group? Why?*

Display it with 5 green cubes as the bottom row.



0 + 5
1 + 4
2 + 3
3 + 2
4 + 1
5 + 0

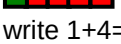
- *Do you see a pattern? Describe the pattern that you see.*

As students describe what they see, write the numerical expression next to the row. Some students may not be ready to connect/make meaning of the numbers, but some will be. If they are not ready, this is not something they will be doing when they create groups of five in their center work.

- *Does anyone see the pattern in a different way?*
- *What's alike about all these groups?* (They all have five cubes/grapes.)
- *What's different about these groups?* (There are different combinations of red and green cubes/grapes. There are different ways to show 5. (equivalence)
- *What happens as the number of green grapes gets bigger?* (The number of red grapes gets smaller.)
- *What operation can we use to find the total number of grapes?*
- *If Leah gave me 2 red grapes and 3 green grapes, I had 5 grapes. I can write the equation 2+3=5. What equation can I write if I have 3 red grapes and 2 green grapes?*
- *If Leah gave me 1 red grape and 4 green grapes, I had 5 grapes. I can write the equation 1+4=5. What equation can I write if I have 4 red grapes?*

What have we learned about "five"?

What will be seen or heard that indicates all students understand the mathematical ideas you intended them to learn?

- There are many ways to make five.
- If 1+4=5, then 4+1=5. I can switch the order of the numbers I am adding together and keep the same sum. (commutative property)
- I can write numbers under the grapes to show how many red grapes plus how many green grapes (numerical expression, e.g., 1+4).
-  One green and 4 red grapes give a total of 5 grapes. So I can write 1+4=5.
- If I take away one red grape, but add one green grape, I still have 5 grapes (compensation).

Ongoing Activities, Day 2 and Beyond

What ongoing activities and centers help to develop and cement students' understanding of *"Five-ness": Five as an Anchor Number*?

Quick Images

Quick images are a powerful way to teach children how to subitize (e.g., when quickly shown a picture of a partially filled in 5-frame, the side of a die or a domino, instantly recognizes the number of dots without counting them). There are three quick image resources (PowerPoint presentations) that complement this lesson posted in the Unit Resources:

- *5-Frames*
- *Dominoes to 5*
- *Dot Cards*

There are suggestions for using each slide in the note section of the slide.
For example, for *5-Frames*:

When shown a quick image (3 seconds or less) of a 5-frame, students identify what they see:

1. *How many frames are filled in with a dot?*
   How many are not filled in?
   How many frames there are in all (5)?
   OR
2. *How many dots do you see?*
   How many more dots are needed to make 5 dots?

Centers

- ***5-Frame***
  After this lesson, students may work independently where they create combinations of five, using two colors of linking cubes. They will be able to determine whether they have found all combinations and explain how they know. Depending on students' readiness, they can record what they have built on a form that has linking cube outlines and write the numerical expression below the combinations.
- ***Computer Applet***
  At an additional center, they may work with a computer applet, *Five Frame*, http://illuminations.nctm.org/ActivityDetail.aspx?id=74. Two games reinforce the concepts in this lesson: "How Many?" and "Fill."

Resources Specific to This Lesson

Both John Van de Walle and Marilyn Burns write about the importance of the 5-frame (and later the 10-frame) as a model for establishing five (and ten) as an anchor number.

Van de Walle, John A., Karp, Karen M., Bay-Williams, Jennifer M. (2009). *Elementary and Middle School Mathematics: Teaching Developmentally, Seventh Edition.* Massachusetts. Allyn and Bacon Publishers.
See Chapter 8, Developing Early Number Concepts and Number Sense. This chapter includes suggestions for quick images (See Dot Plate Flash, page 131).

Islas, Dana. (2010). Five Little Speckled Frogs. *Math Solutions Online Newsletter.* (35) mathsolutions.com
http://www.mathsolutions.com/documents/Five_Speckled_Frogs_i35.pdf

Novakowski, Janice. (2007). Developing "Five-ness" in Kindergarten. *Teaching Children Mathem*atics. 14(4), 226 - 231. Virginia. NCTM Publications. Retrieved September 2011 from
http://www.nctm.org/publications/article.aspx?id=22138

McGrath, Sandra F. and Tammy J. Sanford (2011) Exploring Combinations with Apples and Bananas, *Teaching Children Mathematics*. 18(1), 64. Virginia. NCTM Publications. Retrieved October 2011 from
http://www.nctm.org/publications/article.aspx?id=30522
Kindergarten students explore possible combinations of apples and bananas, sharing insights on combinations and **compensation** (Adding an amount to one number and subtracting the same amount from the other number does not change the total.) Additional student thinking and teacher reflections are provided in an online link provided at this site