Unit: 1 - Power and Polynomial Functions
Algebra II

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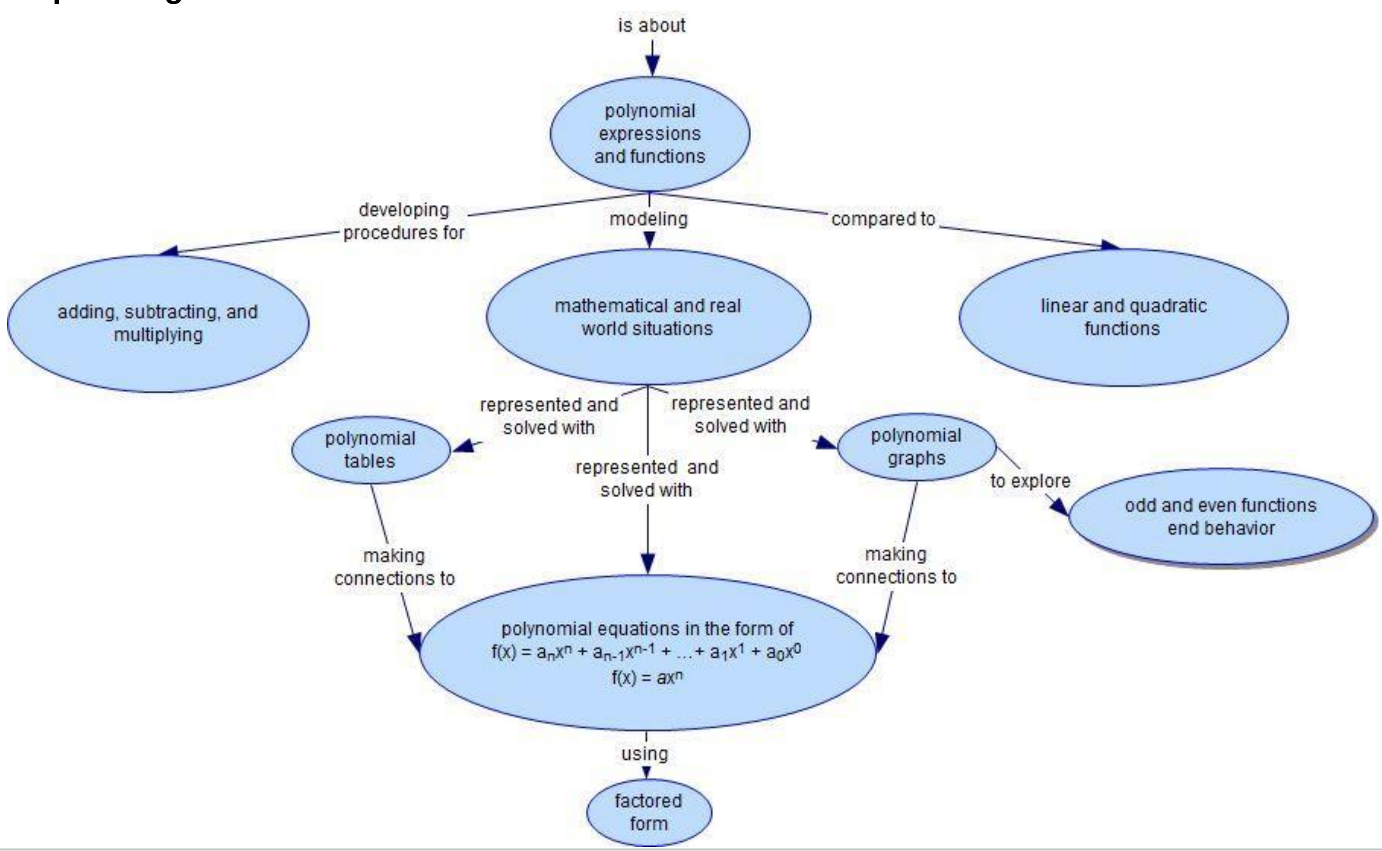
Common Core Initiative

Overarching Questions and Enduring Understandings

How can polynomial functions model real-world or mathematical situations as seen in tables, graphs, and equations that represent these situations?

Graphic Organizer



Unit Abstract

In Algebra 1, students worked to make connections between representations of functions. They observed that different forms of symbolic representations of linear and quadratic functions revealed different features of the functions and vice versa. Factored form, for example, revealed rational zeroes and x-intercepts while vertex form more readily connected to translations of the curves. Students also examined recursive patterns in linear, quadratic, and exponential functions. In this unit they will make connections to these prior experiences as they extend similar reasoning and explorations to power and polynomial functions. Many standards that were introduced in Algebra 1 will be further developed and/or utilized across Algebra 2 units; look for these standards under the Unit Level Standards heading.

Power functions have the general form $f(x)= ax^n$. The study of power functions includes exploring the impact of changing the values of the coefficient a and the exponent n. (In this unit, n is kept to positive whole numbers. In unit 4, the study of these models will include values of n that are negative and fractional, as well as whole numbers.) Numeric patterns in tables will be contrasted to linear and quadratic patterns of change and connected to features in the graphs. As n increases students will also note similarities and differences in shapes of the graphs and examine rotational or reflectional symmetry. Attending to the structures in both the graphs and equations will lead to generalizations about power functions with even exponents compared to power functions with odd exponents. End behavior is introduced as a way to compare and contrast the ranges. Students will use multiple representations to first explore and then generalize that the ranges for odd degree power functions are all real numbers and the ranges for even degree power functions is restricted.

Polynomial functions are the sum of power functions and can be represented symbolically by: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0 x^0$, where n is a whole number, and the coefficients, an, are real numbers. Looking for patterns in tables, graphs, and equations will help develop an understanding of characteristics of odd and even degree polynomial functions as an extension to power functions. These patterns show the relationship of the degree of the polynomial function to the possible number of zeroes and the general shape of the graphs. The key features of the graph and the relationship to the equation and table provide the basis for developing understanding of this broader class of polynomial functions.

Students will also perform arithmetic operations on polynomials. They will rewrite polynomials in factored form to reveal the zeroes using clues from the tables and graphs and structures in equations of the functions. They extend their understanding of the relationship between zeroes and factors from quadratics to polynomial functions. They see that for a polynomial p(x), and a number a, p(a) = 0 if and only if (x-a) is a factor of p(x). (Division by (x-a) is postponed for unit 4 where the Remainder Theorem will be used.) This unit could also include an introduction to the Binomial Theorem and Pascal's Triangle to demonstrate expansion of $(x+y)^n$.

In Algebra 1, students solved systems of linear equations both approximately, with tables and graphs, and exactly using algebraic strategies. They found approximate solutions for systems of equations consisting of both linear and exponential equations. In addition they found both approximate and exact solutions for systems that include linear, exponential and quadratic equations. In this unit, students will extend these concepts to include systems containing polynomial functions.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Algebra

Arithmetic with Polynomials & Rational Functions

HSA-APR.A. Perform arithmetic operations on polynomials.

- HSA-APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

HSA-APR.B. Understand the relationship between zeros and factors of polynomials.

- HSA-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

HSA-APR.C. Use polynomial identities to solve problems.

- HSA-APR.C.5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle.

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the CCSS-M standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

Arithmetic with Polynomials and Rational Expressions

HSA-APR.B. Understand the relationship between zeros and factors of polynomials.

- HSA-APR.B.2. Know and apply the Remainder Theorem: [that]for a polynomial p(x) and a number a, the remainder on division by x – a is p(a),so p(a) = 0 if and only if (x – a) is a factor of p(x).

Developed and/or Utilized Across Multiple Units

Seeing Structures in Expressions

High School: Functions Interpreting Functions **HSF-IF.C. Analyze functions using different representations.** – HSF-IF.C.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Building Functions **HSF-BF.B. Build new functions from existing functions.** • HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	**HSA-SSE.A. Interpret the structure of expressions.** • HSA-SSE.A.1. Interpret expressions that represent a quantity in terms of its context. – HSA-SSE.A.1a. Interpret parts of an expression, such as terms, factors, and coefficients. – HSA-SSE.A.1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. • HSA-SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. **Reasoning with Equations & Inequalities** **HSA-REI.D. Represent and solve equations and inequalities graphically.** • HSA-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). • HSA-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **Interpreting Functions** **HSF-IF.A. Understand the concept of a function and use function notation.** • HSF-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). • HSF-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. • HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
Essential/Focus Questions 1. How do changes in the values of the parameters in a polynomial/power function change the behavior of the graph and/or table? 2. How can the patterns in tables, graphs, and equations help determine the characteristics	**Key Concepts** factors key features of power and polynomial graphs (symmetry, x- and y-intercepts, end behavior) models of polynomial functions (tables, graphs, equations)

(e.g. shape, number of zeros, end behavior, odd/even) of polynomial functions? 3. What are some similarities and differences between polynomial, quadratic, exponential, and linear functions? 4. How are operations with polynomial functions similar to or different from operations with integers? 5. How is solving a system consisting of a linear or a nonlinear equation and a polynomial equation similar/different to solving linear systems and/or quadratic equations?	operations with polynomial functions polynomial function (even, odd, power) polynomial patterns (recursive, common successive difference, explicit) solutions (real or imaginary/complex roots, zeros, x-intercepts) solutions to nonlinear systems
Assessment Tasks Assessment Overview	**Intellectual Processes** **Standards for Mathematical Practice** *Students will have opportunities to:* • **make sense of problems and persevere in solving them** when exploring graph for polynomial functions of various degrees; • **construct arguments and critique the reasoning of others** when discussing key features of odd and even polynomial functions; and • **model with mathematics** to represent contexts that are not well represented by linear, exponential or quadratic functions.
Lesson Sequence Lesson Overview	**Resources** Unit Resources