Unit 3 - Exponential Functions
Algebra I

Friday, December 4, 2015, 9:27AM

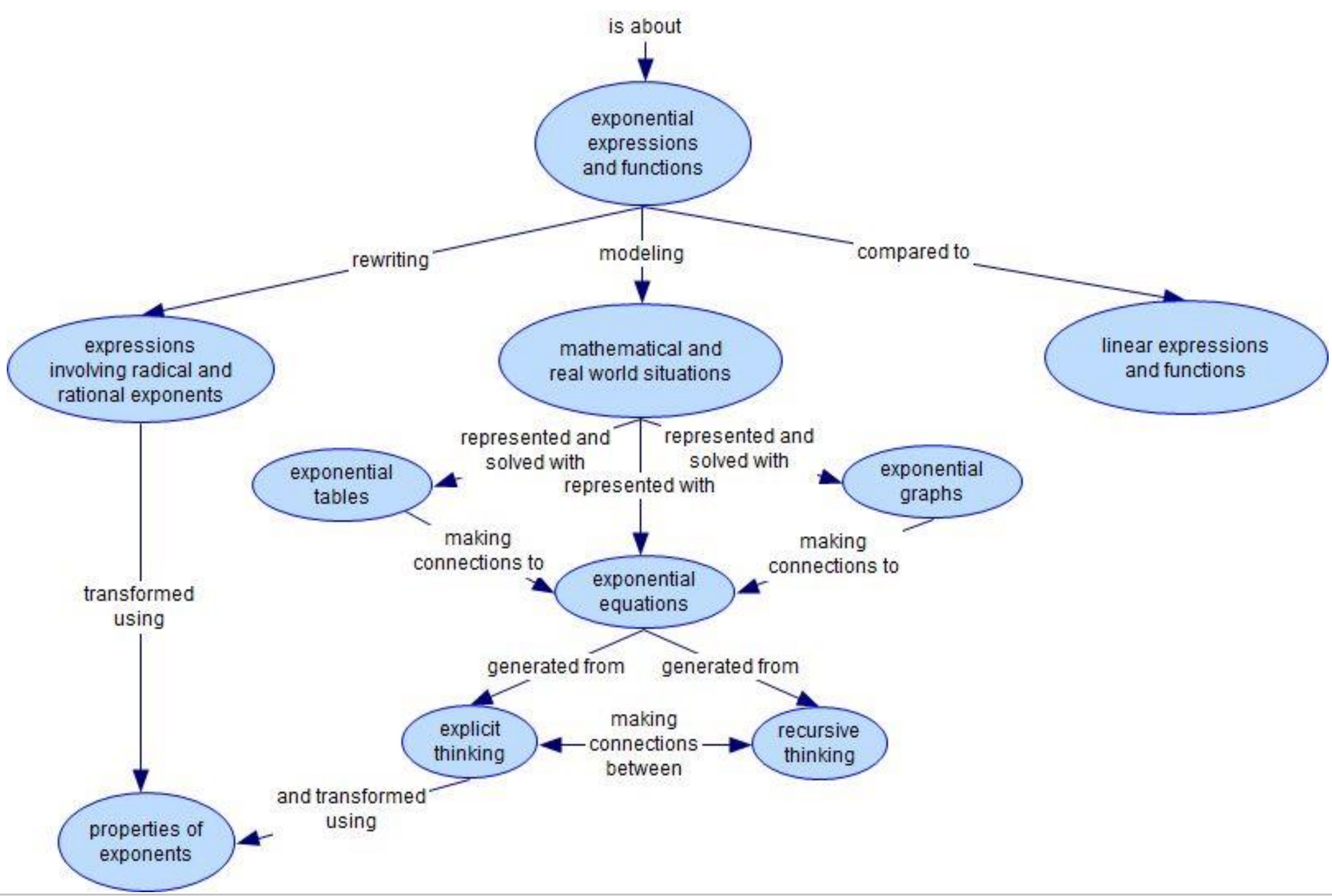
Common Core > 2015-2016 > Grade 9 > Mathematics > Algebra I (**CC**) > Week 13 - Week 18

Common Core Initiative

Overarching Questions and Enduring Understandings

What patterns of change are modeled by exponential functions as seen in real-world situations, and the tables, graphs, and functions rules that represent these situations?

Graphic Organizer



Unit Abstract

In eighth grade, students used recursive thinking and various representations to analyze situations involving exponential patterns of change. This work was primarily done to distinguish nonlinear from linear functions and to give contexts for generating properties of integer exponents. Students used these properties to generate equivalent numerical expressions including those involving scientific notation. The generation of symbolic form, y=ab^x, was not necessarily required of students.

In Algebra 1, students use multiple representations to model exponential functions arising from real-world phenomena like population change, interest on investments, and radioactive decay. Similar to their work with linear functions, students use recursive thinking to generate y-values in a table for exponential functions. They notice that instead of a constant rate of change like linear functions, exponential functions have a constant growth/decay factor.

Students describe recursive thinking with statements like, “to get the NEXT y-value, I multiply the y-value I have NOW by the growth factor of 1.4.” Drawing from recursive thinking, students build proficiency defining exponential functions explicitly with equations that describe the relationship between independent and dependent variables (e.g., f(x)=a*b^x, f(x)=a*b^(x-1), etc. where a and b are non-zero). Eventually, students build facility in writing explicit representations purely from contexts. In order to do so, students also distinguish between rates and factors and learn for example, that a growth rate of 2% equates to a growth factor of 1.02.
Students use and translate among representations to make sense of and problem solve within contexts. Consider the example, a dog receives a 400 mg dose of medicine. Students should see this initial dosage: in the first row of the table when the time column is 0 hours and the medicine column is 400 mg, in the graph at the point (0,400), and in the equation f(x)=400*b^x. In both the table of values and in the graph, students also see the amount of medicine approaching zero making a contextual connection to the concept of horizontal asymptotes. When students are also given the decay rate or factor, they can find the amount of medicine in the dog’s body for any given time. Graphical and tabular representations can be used to estimate the time, given an amount of medicine. (Logarithms are not used to solve exponential equations until Algebra 2.)

Exponent rules facilitate the development of explicit representations from recursive thinking. In addition, they allow students to make efficient comparisons between populations and manipulate algebraic representations to reveal new properties of the relationship being described. (SMP 2 and SMP 7 and MI: Math HSA-SSE.B.3c) http://www.corestandards.org/Math/Practice/ In this work, students must move beyond the Grade 8 rules, generating equivalent numeric expressions with integer exponents, and extend their understanding to applications of rational exponents including rewriting expressions involving radicals and rational exponents using the properties of exponents.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Number/Quantity

The Real Number System

HSN-RN.A. Extend the properties of exponents to rational exponents.

- HSN-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

- HSN-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

High School: Algebra

Seeing Structure in Expressions

HSA-SSE.B. Write expressions in equivalent forms to solve problems.

- HSA-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - HSA-SSE.B.3c. Use the properties of exponents to transform expressions for exponential

Unit Level Standards

There are standards listed in this section for two reasons.

1. The standards have been modified to be appropriate for this unit. Text in gray font is part of the CCSS-M standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.
2. The standards contain content that is developed and/or utilized across multiple units.

Modified For this Unit

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.1.Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations and Inequalities

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.10. Understand that the graph of an functions.HSA-SSE.B.3c. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

High School: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - HSF-IF.C.8b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*

Linear, Quadratic, and Exponential Models

HSF-LE.A. Construct and compare linear and exponential models and solve problems.

- HSF-LE.A.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - HSF-LE.A.1c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- HSA-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models

HSF-LE.A. Construct and compare linear and exponential models and solve problems.

- HSF-LE.A.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- HSF-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF-LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

HSF-LE.B. Interpret expressions for functions in terms of the situation they model.

- HSF-LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.

Developed and/or Utilized Across Multiple Units

Quantities

HSN-Q.A. Reason quantitatively and use units to solve problems.

- HSN-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.
- HSN-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Seeing Structure in Expressions

HSA-SSE.A. Interpret the structure of expressions.

- HSA-SSE.A.1. Interpret expressions that represent a quantity in terms of its context.
  - HSA-SSE.A.1a. Interpret parts of an expression, such as terms, factors, and coefficients.
  - HSA-SSE.A.1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
- HSA-SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Interpreting Functions

HSF-IF.A. Understand the concept of a function and use function notation.

- HSF-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- HSF-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

HSF-IF.B. Interpret functions that arise in applications in terms of the context.

	• HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ • HSF-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ • HSF-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **HSF-IF.C. Analyze functions using different representations.** • HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • HSF-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Building Functions** **HSF-BF.A. Build a function that models a relationship between two quantities.** • HSF-BF.A.1. Write a function that describes a relationship between two quantities. – HSF-BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
Essential/Focus Questions 1. What are some similarities and differences between exponential and linear functions? 2. What similarities do all tables of exponential functions share? 3. How do changes in values of the parameters in an exponential function change the behavior of the graph and/or table? 4. How do you recognize exponential growth or decay from a graph, function rule, table, or real-world situation? 5. How can understanding rules of exponents assist in representing and interpreting exponential functions from specific contexts?	**Key Concepts** asymptotic behavior exponential growth and decay exponential patterns (recursive and explicit) growth/decay factor models of exponential functions (tables, graphs, equations) properties of exponents

Assessment Tasks

Assessment Overview
Assessment Task
Independent Practice
Professional Learning Task - Student Misconception Video Discussion

Intellectual Processes

Standards for Mathematical Practice

Students will have opportunities to:

- **attend to precision** when giving answers that represent very large or very small values; and
- **look for and express regularity in repeated reasoning** as seen in tables to express mathematical ideas regarding exponential patterns of change precisely;
- **use appropriate tools strategically** when selecting, applying, and translating among mathematical representations to solve exponential equations.

Lesson Sequence

Lesson Overview
Student Handout - POSTED with permission
Professional Learning Task - Thinking about the lesson

Resources

Unit Resources