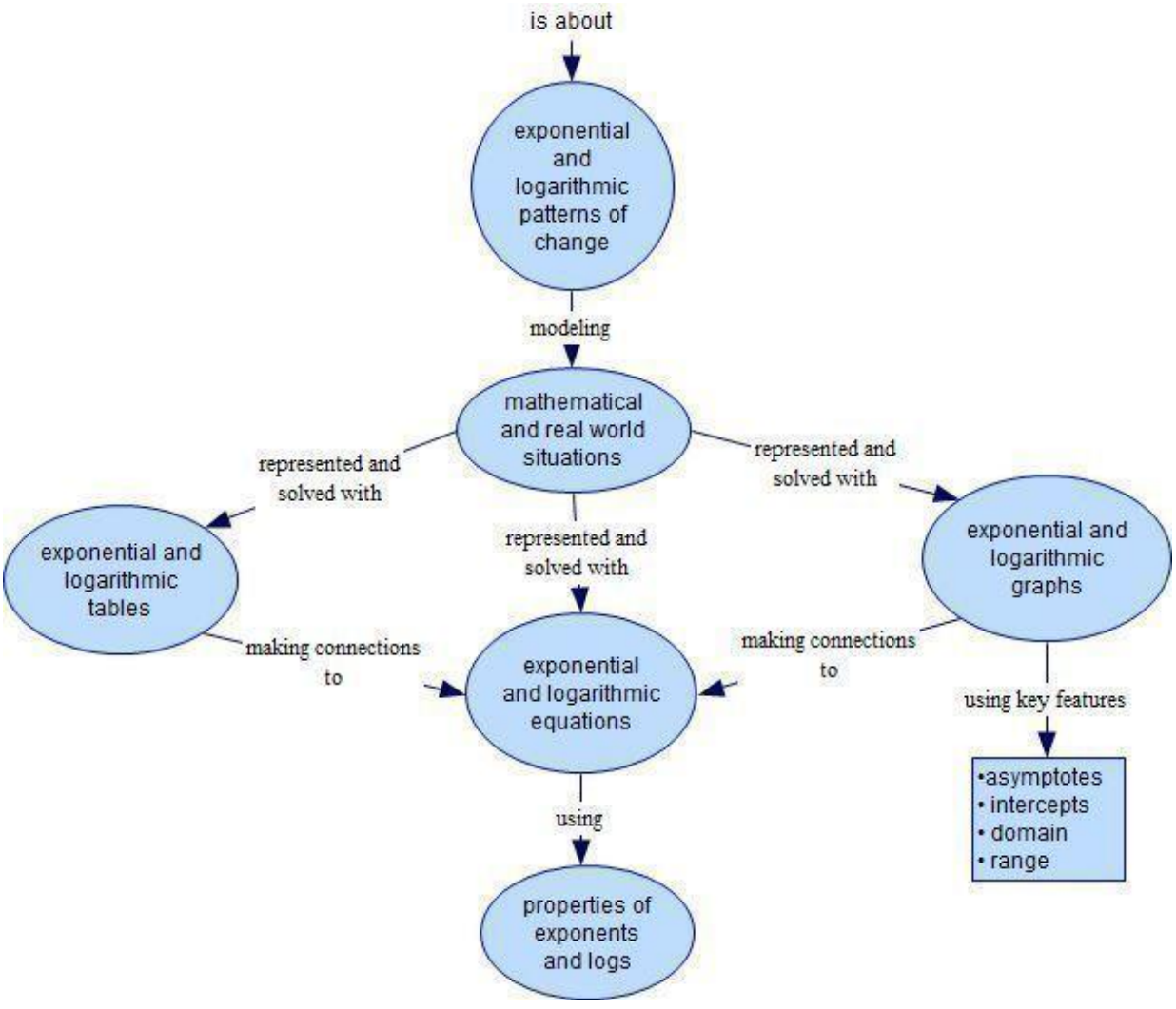
Common Core Initiative

Overarching Questions and Enduring Understandings

What is the connection between exponential and logarithmic functions? What patterns of change are modeled by logarithmic functions as seen in real-world situations, and the tables, graphs, and functions rules that represent these situations?

Graphic Organizer



Unit Abstract

The study of exponential functions begins in grade 8 and is a key component of the Algebra I curriculum. In Algebra II, students use their understanding of exponential functions with the definition of inverse functions to explore logarithms. The concept of function inverses is linked to composition of functions. Introducing and using composition of functions in this unit provides the opportunity to verify whether one function is the inverse of another. Recognizing

the inverse relationship between exponential and logarithmic functions will help students to understand the definition of a logarithm; to know and be able to use the properties of logarithms; to make graphical connections between the two functions; and to solve exponential equations not only by using graphs and tables, but also the properties of logarithms. Connections to real-world situations are found in looking at situations that use a logarithmic scale to report values such as the Richter scale, the pH scale and the measurement of sound intensity using a decibel scale.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- HSF-IF.C.8b. Use the properties of exponents to interpret expressions for exponential functions.

For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Building Functions

HSF-BF.A. Build a function that models a relationship between two quantities.

- HSF-BF.A.1b. Combine standard function types using arithmetic operations.

For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

- HSF-BF.A.1c. (+) Compose functions.

For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.4b. (+) Verify by composition that one function is the inverse of another.
- HSF-BF.B.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models

HSF-LE.A. Construct and compare linear and exponential models and solve problems.

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

Linear, Quadratic, and Exponential Models

HSF-LE.A. Construct and compare linear and exponential models and solve problems.

- HSF-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Developed and/or Utilized Across Multiple Units

Quantities

HSN-Q.A. Reason quantitatively and use units to solve problems.

- HSN-Q.A.2. Define appropriate quantities for the purpose of descriptive modeling.
- HSN-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

- HSF-LE.A.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Essential/Focus Questions

1. How can the properties of logarithms be used to write algebraic expressions in equivalent forms?
2. What types of real world relationships are best described using a logarithmic scale? Why?
3. What relationships - graphical, algebraic, numeric - exist between a function and its inverse?
4. Why can't a logarithm have an argument of zero or a negative number?
5. What are the similarities and differences between exponential and logarithmic functions?

Key Concepts

asymptote
base of a logarithm
base ten logarithms (common logarithms)
composition of functions
domain
e
end behavior
exponential function
exponential models (compound interest, populations, radioactivity)
$f(x) - e^x$
$f(x) = ab^x$
inverse function
logarithmic function
logarithmic scales (Richter scale for earthquakes, decibel for acoustic power, entropy, pH for acidity, stellar magnitude scale for brightness of stars)
$\log_b x = y$
natural logarithms
properties of exponents
properties of logarithms
range
transformation of functions

Assessment Tasks

Assessment Overview
Student Handouts

Intellectual Processes

Standards for Mathematical Practice

Students will have opportunities to:

- **make sense of problems and persevere in solving them:** apply and adapt a variety of appropriate strategies to solve exponential and logarithmic functions;
- **model with mathematics**: recognize and apply mathematics in contexts outside of mathematics; and
- **use appropriate tools strategically**: select, apply, and translate among mathematical representations of exponential and logarithmic functions to solve problems
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Lesson Sequence

Lesson Overview
Highlight Lesson Handout (CPMP text)
Optional Lesson Formative Assessment

Resources

Unit Resources
NCTM Illuminations Website