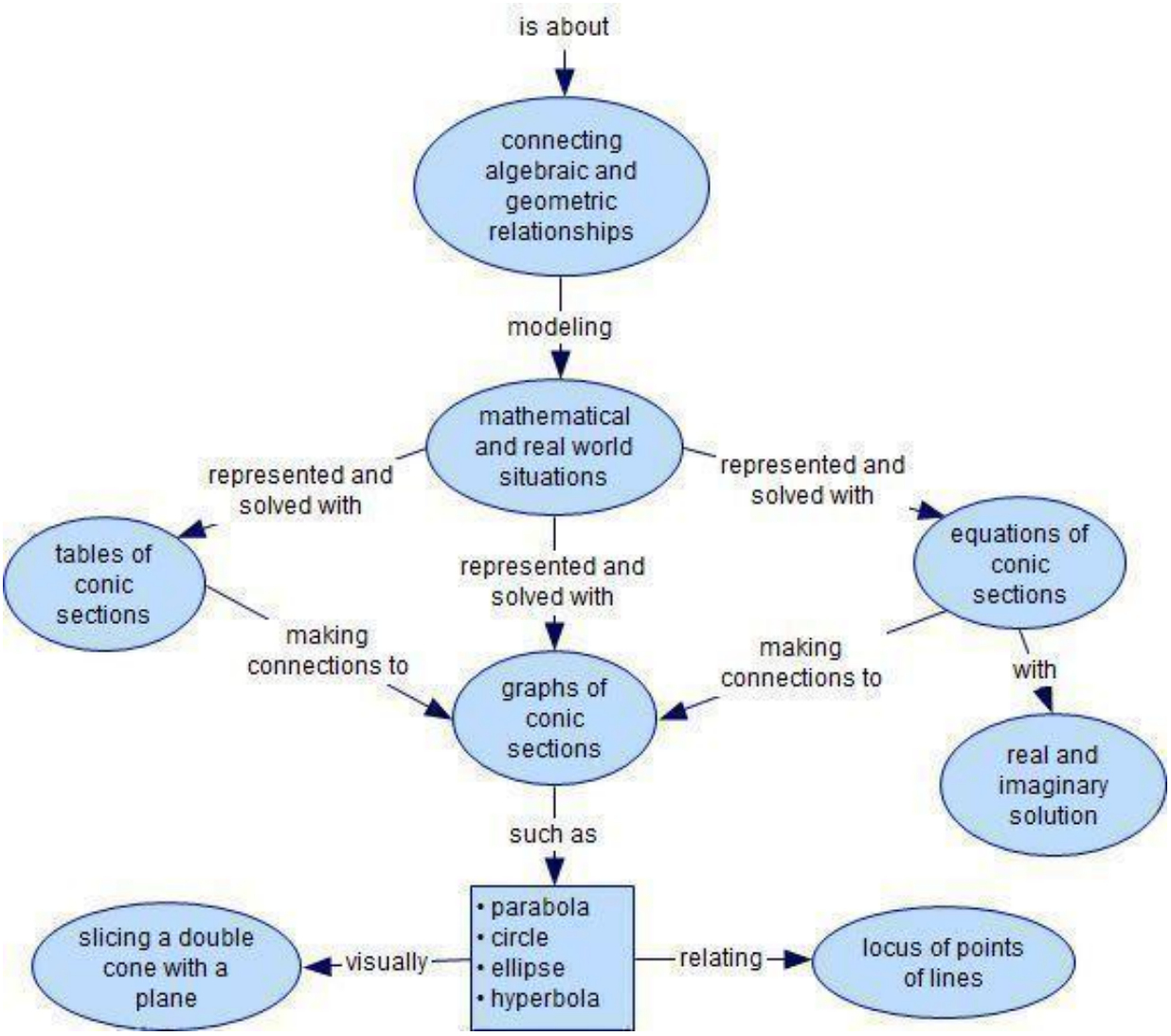
Common Core Initiative

Overarching Questions and Enduring Understandings

How can algebraic and geometric ideas be used to explore and connect representations of quadratic relations from the conic sections?

Graphic Organizer



Unit Abstract

This unit is an extension of the Quadratic Units from Algebra 1. Students will build on prior experiences with quadratic functions (parabolas) graphing, factoring, completing the square, solving and transforming parabolas graphically and algebraically this time encountering equations with imaginary solutions (no real solutions) since not all solutions to quadratic equations are real numbers. Therefore, the study of complex numbers will be included in this unit. They will also explore quadratic relations, more commonly known as conic sections, by making connections

between algebraic and geometric relationships.

Conic sections are all considered quadratic curves because the highest power of any variable is two. Slicing a double napped cone (two cones with a shared apex) with planes at various angles will yield a parabola, circle, ellipse, or hyperbola. Students explore the locus of points that determine the four conic sections and use the distance formula along with geometric descriptions to generate equations for each of the conic sections. Observing the equations (both standard and transformation form) and the resulting graphs should include identifying type of conic section shown and all of its special properties.

Unit Overview (Word)

Unit Overview (PDF)

Content Expectations/Standards

High School: Number/Quantity

The Complex Number System

HSN-CN.A. Perform arithmetic operations with complex numbers.

- HSN-CN.A.1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.
- HSN-CN.A.2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- HSN-CN.A.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

HSN-CN.C. Use complex numbers in polynomial identities and equations.

- HSN-CN.C.7. Solve quadratic equations with real coefficients that have complex solutions.
- HSN-CN.C.8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
- HSN-CN.C.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

High School: Algebra

Seeing Structure in Expressions

HSA-SSE.B. Write expressions in equivalent forms to solve problems.

- HSA-SSE.B.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
- HSA-SSE.B.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Reasoning with Equations & Inequalities

HSA-REI.B. Solve equations and inequalities in one variable.

Unit Level Standards

There are standards listed in this section for two reasons.

1. ***The standards have been modified to be appropriate for this unit. Text in gray font is part of the Michigan K-12 standard but does not apply to this unit. Text in brackets denotes a modification that has been made to the standard.***
2. ***The standards contain content that is developed and/or utilized across multiple units.***

Modified For this Unit

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Developed and/or utilized across multiple units

Seeing Structure in Expressions

HSA-SSE.B. Write expressions in equivalent forms to solve problems.

- HSA-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Creating Equations

HSA-CED.A. Create equations that describe numbers or relationships.

- HSA-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Reasoning with Equations & Inequalities

HSA-REI.D. Represent and solve equations and inequalities graphically.

- HSA-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- HSA-REI.B.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- HSA-REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

HSA-REI.C. Solve systems of equations.

- HSA-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$.

High School: Functions

Interpreting Functions

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSF-IF.C.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

High School: Geometry

Expressing Geometric Properties with Equations

HSG-GPE.A. Translate between the geometric description and the equation for a conic section

- HSG-GPE.A.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- HSG-GPE.A.2. Derive the equation of a parabola given a focus and directrix.
- HSG-GPE.A.3. (+) Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola.

Geometric Measurement & Dimension

HSG-GMD.B. Visualize the relation between two-dimensional and three-dimensional objects

- HSG-GMD.B.4. Identify cross-sectional shapes of slices of three-dimensional objects, and identify three-

Interpreting Functions

HSF-IF.A. Understand the concept of a function and use function notation.

- HSF-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

HSF-IF.B. Interpret functions that arise in applications in terms of the context.

- HSF-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
- HSF-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
- HSF-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

HSF-IF.C. Analyze functions using different representations.

- HSF-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- HSF-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- HSF-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

HSF-BF.A. Build a function that models a relationship between two quantities.

- HSF-BF.A.1. Write a function that describes a relationship between two quantities.

HSF-BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

- HSF-BF.A.1b. Combine standard function types

dimensional objects generated by rotations of two-dimensional objects.

using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

HSF-BF.B. Build new functions from existing functions.

- HSF-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Essential/Focus Questions

1. What shapes result from passing a plane through a cone?
2. How are the equation and the properties of a circle and ellipse similar and different?
3. Given the equation of a conic section, how can you identify whether the graph will be a circle, ellipse, parabola, or hyperbola?
4. How can you use a line for the directrix and a point for the focus to sketch a resulting parabola?
5. What methods are available to convert an equationin the general conic form into a form more suitable for graphing?

Key Concepts

relationship of circles, ellipses, and hyperbolas to cones
circle
ellipse
parabola
hyperbola
locus of points
completing the square
discriminant
symmetry, lines and axes of graphs of conic sections
major axis andminor axis
transverse axis and conjugate axis
asymptotes
focus, foci
vertex, vertices
distance formula
directrix
eccentricity
$i = \sqrt{-1}$

complex number (a + bi, where a and b are Real Numbers)
conjugate of a complex number

Standard form
$Ax^2 + Bxy + Cy^2 + Dx+Ey + F = 0$

Transformation form
$(x-h)^2+ (y-k)^2= r^2$

$\frac{(x-h)^2}{a^2}+ \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$x = a(y-k)^2+ h$ or $y = a(x-h)^2+k$

Assessment Tasks	Intellectual Processes
Assessment Overview See the Light! See The Light, Again!	**Standards for Mathematical Practice** *Students have opportunities to:* • **make sense of problems and persevere in solving them** by building new knowledge of conic sections through problem solving; • **model with mathematics** to develop an understanding of how mathematical ideas interconnect and build on one another to produce a coherent whole; and • **use appropriate tools strategically** particularly technology based tools to recognize and use connections among mathematical ideas.
Lesson Sequence	**Resources**
Lesson Overview	Unit Resources